Nongaussian and nonscale-invariant perturbations from tachyonic preheating in hybrid inflation

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We show that in hybrid inflation it is possible to generate large second-order perturbations in the cosmic microwave background due to the instability of the tachyonic field during preheating. We carefully calculate this effect from the tachyon contribution to the gauge-invariant curvature perturbation, clarifying some confusion in the literature concerning nonlocal terms in the tachyon curvature perturbation; we show explicitly that such terms are absent. We quantitatively compute the nongaussianity generated by the tachyon field during the preheating phase and translate the experimental constraints on the nonlinearity parameter f_{NL} into constraints on the parameters of the model. We also show that nonscale-invariant second-order perturbations from the tachyon field with spectral index $n = 4$ can become larger than the inflaton-generated first-order perturbations, leading to stronger constraints than those coming from nongaussianity. The width of the excluded region in terms of the logarithm of the dimensionless coupling g, grows linearly with the log of the ratio of the Planck mass to the tachyon VEV, $\log(M_p/v)$; hence very large regions are ruled out if the inflationary scale v is small. We apply these results to string-theoretic brane-antibrane inflation, and find a stringent upper bound on the string coupling, $g_s < 10^{-4.5}$.

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I. INTRODUCTION

Inflation [1] has become the dominant paradigm for the generation of the anisotropies of the Cosmic Microwave Background (CMB) (see [2, 3] for a review). During inflation quantum fluctuations are redshifted out of the horizon where they become "frozen" and later re-enter the horizon after Big Bang Nucleosynthesis. Observation of the temperature anisotropies of the CMB directly constrain the curvature perturbation somewhat before cosmological scales re-enter the horizon. Current observations show that the curvature perturbation is gaussian with a scale-invariant spectrum to within experimental limits, which is consistent with the predictions of inflation. Nevertheless, some nongaussianity is expected to be generated during inflation [4]-[8] (see [9] for a review). It is interesting to study the level of nongaussianity, usually parameterized by a dimensionless nonlinearity parameter f_{NL}, in various scenarios for inflation, since it can be an important way to discriminate between different models. For example, the curvaton mechanism [10] for the generation of cosmological perturbations may be ruled out by future non-observation of nongaussianity [11] (see, however, [12]).

One way to generate significant levels of nongaussianity at the end of inflation is by preheating [13]-[19]. Preheating refers to the nonperturbative production of particles which arises either due to coherent oscillations [20] or tachyonic (spinodal) instability [22]-[24] (see also [25, 26]). It has also been suggested that preheating might affect the first order variation of the curvature perturbation, perhaps dominating the contribution of the inflaton and thus modifying the standard predictions of inflation [27]-[30].

In this paper we carefully reconsider the generation of nongaussian fluctuations from tachyonic preheating in hybrid inflation [31] using second order cosmological perturbation theory similarly to [6, 32]. See [33] for constraints on the parameter space of hybrid inflation coming from the WMAP data. During hybrid inflation the inflaton field φ is displaced from its preferred value, and inflation is driven by the false vacuum energy of the "waterfall" or tachyon field σ which is trapped in its false vacuum by interactions with the inflaton. When the inflaton reaches some critical value the tachyon effective mass squared becomes negative and its fluctuations grow exponentially due to the spinodal instability. The exponential growth continues until the fluctuations of the tachyon start to oscillate about the true vacuum. At this stage the tachyon fluctuations become nonperturbative and their back-reaction brings inflation to an end. Here we study the evolution of the second-order curvature perturbation due to the tachyonic instability and find that it can lead to a large level of nongaussianity, if the tachyon was lighter than the Hubble scale $m_\sigma < 3H/2$, during a sufficient part of the inflationary epoch. (If $m_\sigma \gg H$, the fluctuations of σ are exponentially suppressed during inflation.) In the hybrid inflation model, it can be natural to have $m_\sigma < 3H/2$ during inflation, since m_σ^2 changes sign at the end of inflation. If the inflaton is rolling slowly enough, m_σ^2 will naturally already be close to zero even before the end of inflation, at the time of horizon crossing.

The scenario is summarized as follows. During inflation the squared mass of the tachyon field varies linearly with the number of e-foldings of inflation, in the vicinity of the point where it passes through zero: $m_\sigma^2 = cH^2N$, taking $N = 0$ to be the value where $m_\sigma^2 = 0$, and c is a

function of the parameters of hybrid inflation. Inflation starts at some $N \equiv N_i < 0$, and ends at $N \equiv N_* > 0$, for a total of $N_e = -N_i + N_*$ e-foldings. We are going to limit our inquiry to regions of parameter space where the linear behavior of m_σ^2 is valid during the full duration of inflation, since this technical assumption makes the calculations tractable; however we will show that this assumption must often be satisfied anyway, as a consequence of the experimental limit on the inflationary spectral index. At $N = 0$, fluctuations of the tachyon field start to grow exponentially, and can make a large contribution to the curvature perturbation at second order in cosmological perturbation theory, resulting in a scale-noninvariant component both in the CMB temperature power spectrum, and its nongaussian correlations. Our goal is to derive constraints on the parameter space of hybrid inflation from these effects.

We start in section II by defining the perturbations up to second order in the metric and inflationary fields. In section III the hybrid inflation model is reviewed, starting with the dynamics of the fields at zeroth order, and then their first order perturbations, with emphasis on the dynamics of the tachyon field fluctuations toward the end of inflation. In section IV we solve for the second order curvature perturbation which is induced by the tachyonic fluctuations. This is used in section V to compute the bispectrum (three-point function) and the nonlinearity parameter f_{NL}, as well as contributions to the spectrum itself. In section VI we incorporate experimental constraints on nongaussianity as well as on the spectrum, to derive excluded regions in the parameter space of the hybrid inflation model. In section VII we consider the possibility of generating significant nongaussianity in brane inflation. We conclude in section VIII, comparing our results with previous treatments. We give technical details concerning mode functions in de Sitter space, the second order perturbed Einstein equations, the inflaton curvature perturbation, Fourier transforms of convolutions, the construction of the tachyon curvature perturbation and the approximation of an adiabatically varying tachyon mass in appendices A-F.

II. METRIC AND MATTER PERTURBATIONS

In this section we write down the perturbations of the metric and matter fields about a spatially flat Robertson-Walker background following [6]. Greek indices run over the full spacetime $\mu, \nu = 0, 1, 2, 3$ while latin indices run only over the spatial directions $i, j = 1, 2, 3$. We will work mostly in conformal time τ, related to cosmic time t by $dt = a d\tau$. Differentiation with respect to conformal time will be denoted by $f' = \partial_\tau f$ and with respect to cosmic time by $\dot{f} = \partial_t f$.

The metric is expanded up to second order in fluctua-

tions as

$$g_{00} = -a(\tau)^2 \left[1 + 2\phi^{(1)} + \phi^{(2)} \right] \tag{1}$$

$$g_{0i} = a(\tau)^2 \left[\partial_i \omega^{(1)} + \frac{1}{2} \partial_i \omega^{(2)} + \omega_i^{(2)} \right] \tag{2}$$

$$g_{ij} = a(\tau)^2 \left[(1 - 2\psi^{(1)} - \psi^{(2)}) \delta_{ij} + D_{ij}(\chi^{(1)} + \frac{1}{2}\chi^{(2)}) \right.$$
$$\left. + \frac{1}{2}(\partial_i \chi_j^{(2)} + \partial_j \chi_i^{(2)} + \chi_{ij}^{(2)}) \right] \tag{3}$$

where $D_{ij} = \partial_i \partial_j - \frac{1}{3}\delta_{ij}\partial^k \partial_k$ is a trace-free operator. The fluctuations are decomposed such that the vector perturbations are transverse $\partial^i \omega_i^{(2)} = \partial^i \chi_i^{(2)} = 0$ while the tensor perturbations are transverse, traceless and symmetric: $\partial^i \chi_{ij}^{(2)} = 0$, $\chi_i^{i(2)} = 0$, $\chi_{ij}^{(2)} = \chi_{ji}^{(2)}$. In the above we have neglected the vector and tensor perturbations at first order, which are small, since vector perturbations decay with time, while tensors are suppressed by the slow roll parameter ϵ. The same is not true at second order, however, since the second order tensors and vectors are sourced by the first order scalar perturbations. We adopt the generalized longitudinal gauge defined by $\omega^{(1)} = \omega^{(2)} = \omega_i^{(2)} = 0$ and $\chi^{(1)} = \chi^{(2)} = 0$. The metric in this gauge becomes

$$g_{00} = -a(\tau)^2 \left[1 + 2\phi^{(1)} + \phi^{(2)} \right] \tag{4}$$

$$g_{0i} = 0 \tag{5}$$

$$g_{ij} = a(\tau)^2 \left[(1 - 2\psi^{(1)} - \psi^{(2)}) \delta_{ij} \right.$$
$$\left. + \frac{1}{2}(\partial_i \chi_j^{(2)} + \partial_j \chi_i^{(2)} + \chi_{ij}^{(2)}) \right]. \tag{6}$$

In hybrid inflation the matter content consists of two scalar fields which are expanded in perturbation theory as

$$\varphi(\tau, \vec{x}) = \varphi_0(\tau) + \delta^{(1)}\varphi(\tau, \vec{x}) + \frac{1}{2}\delta^{(2)}\varphi(\tau, \vec{x}) \tag{7}$$

$$\sigma(\tau, \vec{x}) = \sigma_0(\tau) + \delta^{(1)}\sigma(\tau, \vec{x}) + \frac{1}{2}\delta^{(2)}\sigma(\tau, \vec{x}). \tag{8}$$

where φ is the inflaton and σ the tachyon (or "waterfall" field). In hybrid inflation the time-dependent vacuum expectation value (VEV) of the tachyon field is set to zero $\sigma_0(\tau) = 0$, about which we will say more later.

The perturbations are defined so that $\langle \delta^{(i)}\varphi \rangle = 0$, hence $\langle \varphi(\tau, \vec{x}) \rangle = \varphi_0(\tau)$. At first order in perturbation theory this is automatic since $\delta^{(1)}\varphi$ contains only one annihilation/creation operator. However, at higher order in perturbation theory the homogeneous $k = 0$ mode of the fluctuation must be subtracted by hand in order to ensure that all of the zero mode of the field is described by the nonperturbation background.

The Einstein tensor and stress-energy tensor expanded up to second order in perturbation theory can be found in [32]. We do not reproduce these results here, but we have carefully checked all the results from [32] which are relevant for our analysis.

III. HYBRID INFLATION

We consider hybrid inflation in which both the inflaton and the tachyon are real fields with the potential

$$V(\varphi, \sigma) = \frac{\lambda}{4} \left(\sigma^2 - v^2 \right)^2 + \frac{m_\varphi^2}{2} \varphi^2 + \frac{g^2}{2} \varphi^2 \sigma^2. \quad (9)$$

This potential will give rise to topological defects at the end of inflation—domain walls in the σ field—which could produce large nongaussianities apart from the ones which we consider ([34]). But these domain walls must somehow be rendered unstable to avoid overclosure of the universe, for example through the addition of a small term like $\mu \sigma^3$ to (9). We will ignore this issue here. It would be interesting to consider a complex tachyon field [67] whose defects formed at the end of inflation are cosmic strings, which are much more phenomenologically viable than domain walls.

A. Background Dynamics

At the homogeneous level, the usual Friedmann and Klein-Gordon equations for the scale factor and the matter fields are

$$3H^2 = \frac{\kappa^2}{2} \left(\dot{\varphi}_0^2 + \dot{\sigma}_0^2 \right) + \kappa^2 V, \quad (10)$$

$$0 = \ddot{\varphi}_0 + 3H\dot{\varphi}_0 + \frac{\partial V}{\partial \varphi} \quad (11)$$

$$0 = \ddot{\sigma}_0 + 3H\dot{\sigma}_0 + \frac{\partial V}{\partial \sigma} \quad (12)$$

where $\kappa^2 = M_p^{-2} = 8\pi G_N$. Here and elsewhere the potential and its derivatives are understood to be evaluated on the background values of the fields so that $V = V(\varphi_0, \sigma_0)$ and $\partial V/\partial \varphi = \partial V/\partial \varphi|_{\{\varphi = \varphi_0, \sigma = \sigma_0\}}$, for example. For the potential (9) we have $\partial V/\partial \sigma = \partial^2 V/\partial \sigma \partial \varphi = 0$, provided that $\sigma_0 = 0$. We will apply this simplification to all subsequent results.

To see why one should set $\sigma_0(\tau) = 0$, notice that the tachyon effective mass is $m_\sigma^2 = g^2 \varphi_0^2 - \lambda v^2 > 0$. That is, the tachyon mass squared starts out being positive during inflation. Provided that there was a long enough prior period of inflation, any initial departure of σ from zero would be exponentially damped. At some point, m_σ^2 becomes negative, and the tachyonic instability begins. However, it is still true that $\sigma_0(\tau)$ remains zero even then, since the universe will consist of equal numbers of domains with $\sigma > 0$ and $\sigma < 0$. On average, these give zero, which is the definition of the zeroth order field σ_0. The departures of σ from zero between domain walls which form are taken account in the fluctuations of the field. Thus it is consistent to set $\sigma_0(\tau) = 0$ for all times in our analysis. (Notice that the situation would be different if we considered a complex scalar field σ in which case the phase transition would lead to cosmic string formation and the radial degree of freedom $|\sigma|$ would not average to zero at late times.)

We will make extensive use of the slow roll parameters, defined by

$$\epsilon = \frac{\kappa^2}{2} \frac{\dot{\varphi}_0^2}{H^2} = -\frac{\dot{H}}{H^2} \cong \frac{M_p^2}{2V^2} \left(\frac{\partial V}{\partial \varphi} \right)^2,$$

$$\epsilon - \eta = \frac{\ddot{\varphi}_0}{H\dot{\varphi}_0} \cong \epsilon - \frac{M_p^2}{V} \left(\frac{\partial^2 V}{\partial \varphi^2} \right)$$

so that, during inflation

$$\eta \cong 4 \frac{M_p^2 \, m_\varphi^2}{\lambda v^4}, \qquad \epsilon \cong 8 \left(\frac{M_p \, m_\varphi^2 \, \varphi_0}{\lambda v^4} \right)^2 \quad (13)$$

Notice that if $m_\varphi^2 \varphi_0^2 \ll \lambda v^4$, then $\epsilon \ll \eta$. This is equivalent to demanding that the false vacuum energy of the tachyon dominates during inflation, which is the assumption usually made for hybrid inflation:

$$V(\varphi_0, \sigma_0 = 0) = \frac{\lambda v^4}{4} + \frac{m_\varphi^2}{2} \varphi_0^2 \cong \frac{\lambda v^4}{4}.$$

During the slow roll phase the inflaton equation of motion $3H\dot{\varphi}_0 + m_\varphi^2 \varphi_0 \cong 0$ has solution

$$\varphi_0(t) = \varphi_s \exp\left(-\frac{m_\varphi^2(t - t_s)}{3H} \right) = \varphi_s \left(\frac{a(t)}{a_s} \right)^{-\eta} \quad (14)$$

where we used $a(t) = a_s e^{H(t - t_s)}$, with t_s an arbitrary time. The Hubble scale remains approximately constant, $3H^2 \cong \lambda v^4/(4M_p^2)$. Since φ_0 is decreasing and $\sigma_0 = 0$, the slow roll parameter ϵ actually decreases slowly during inflation while η remains constant.

B. Inflationary Dynamics of First Order Fluctuations

Having argued that the background inflaton field φ_0 is described by (14) with H approximately constant during both inflation and the tachyonic instability phase, we now briefly discuss the dynamics of the first order metric and inflaton fluctuations for $\epsilon, \eta \ll 1$. This section is largely review since when $\sigma_0 = 0$ the first order metric and inflaton perturbations obey exactly the same equations as in single field inflation; however, we include some details of the calculations since similar equations will arise when we study the second order metric fluctuations.[1] We work in conformal time which is most convenient for the ensuing calculations.

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[1] As will be shown, the second order metric fluctuation $\phi^{(2)}$ obeys an inhomogeneous equation where the differential operator is identical to the one which determines the dynamics of $\phi^{(1)}$; thus an understanding of the first order solutions simplifies the construction of the Green function for the second order fluctuations.

The $\delta^{(1)}G^i_j = \kappa^2\delta^{(1)}T^i_j$ Einstein equation for $i \neq j$ implies that $\phi^{(1)} = \psi^{(1)}$ which is a well known result. We apply this simplification in the following. The $\delta^{(1)}G^0_i = \kappa^2\delta^{(1)}T^0_i$ Einstein equation is a constraint

$$\phi'^{(1)} + \mathcal{H}\phi^{(1)} = \frac{\kappa^2}{2}\varphi'_0\,\delta^{(1)}\varphi \qquad (15)$$

which means that the first order metric perturbation $\phi^{(1)}$ and the first order inflaton perturbation $\delta^{(1)}\varphi$ are not independent. Once either $\phi^{(1)}$ or $\delta^{(1)}\varphi$ is known, the other may be computed from (15), though it is simplest to solve for $\phi^{(1)}$ and use (15) to compute $\delta^{(1)}\varphi$. One obtains a dynamical equation for the metric perturbation by applying (15) to the sum of $\delta^{(1)}G^0_0 = \kappa^2\delta^{(1)}T^0_0$ and $\delta^{(1)}G^i_i = \kappa^2\delta^{(1)}T^i_i$ Einstein equations. The result is

$$\phi''^{(1)}_k + 2\left(\mathcal{H} - \frac{\varphi''_0}{\varphi'_0}\right)\phi'^{(1)}_k + \left[2\left(\mathcal{H}' - \mathcal{H}\frac{\varphi''_0}{\varphi'_0}\right) + k^2\right]\phi^{(1)}_k = 0. \qquad (16)$$

Notice that the perturbed Klein-Gordon equation for the inflaton is not needed to close the system of equations. We discuss the slow roll solutions of (16) in some detail since later on we will need to construct the Green function for the operator in the left-hand-side of (16).

The conformal time slow roll parameters are

$$\epsilon = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} = \frac{\kappa^2}{2}\frac{\varphi'^2_0}{\mathcal{H}^2}, \qquad (17)$$

$$\epsilon - \eta = \frac{\varphi''_0}{\mathcal{H}\varphi'_0} - 1. \qquad (18)$$

During a pure deSitter phase $\epsilon = \eta = 0$ and the scale factor evolves as $a(\tau) = -1/(H\tau)$ with H constant. During inflation, however, the Hubble scale evolves slowly as (17) so that for small ϵ one has [2]

$$a(\tau) = -\frac{1}{H\tau}\frac{1}{1-\epsilon}$$

and

$$\mathcal{H} = \frac{a'}{a} = \dot{a} = aH = \frac{-1}{\tau(1-\epsilon)}. \qquad (19)$$

Note that during a slow roll phase we can treat ϵ and η as constant (even though H is not exactly constant) since ϵ', η' are second order in slow roll parameters:

$$\epsilon' = -2\epsilon(\eta - 2\epsilon)\mathcal{H} \ll \epsilon\mathcal{H}, \qquad \eta' = 2\epsilon\,\eta\mathcal{H} \ll \eta\mathcal{H}.$$

The above equality for η' is only strictly correct if $\partial^3 V/\partial\varphi^3 = 0$ which is true in the case of interest. The statement that the slow roll parameters are approximately constant is, however, quite general.[2]

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[2] Of course this statement generalizes to cosmic time as well.

The dynamical equation for $\phi^{(1)}$ (16) can be rewritten in terms of the slow roll parameters as

$$\phi''^{(1)}_k - \frac{2}{\tau}(\eta - \epsilon)\phi'^{(1)}_k + \left[\frac{2}{\tau^2}(\eta - 2\epsilon) + k^2\right]\phi^{(1)}_k = 0 \quad (20)$$

where we used (19) and dropped higher order terms in ϵ, η. Treating the slow roll parameters as constant, the equation (20) has an exact solution

$$\phi^{(1)}_k(\tau) = (-\tau)^{1/2+\eta-\epsilon}\left[c_1(k)H^{(1)}_\nu(-k\tau) + c_2(k)H^{(2)}_\nu(-k\tau)\right] \qquad (21)$$

where $\nu \cong 1/2 + (3\epsilon - \eta)$ to lowest order in slow roll parameters.

It remains to fix the coefficients $c_1(k)$, $c_2(k)$ in (21). The variable in terms of which the action is canonically normalized is the Mukhanov variable [3]

$$V^{(1)}_k = a\left[\delta^{(1)}\varphi_k + \frac{\varphi'_0}{\mathcal{H}}\phi^{(1)}_k\right]$$

where the inflaton fluctuation is solved for using the constraint equation (15). We fix $c_1(k)$, $c_2(k)$ by requiring that $V^{(1)}_k \cong e^{-ik\tau}/\sqrt{2k}$ on small scales $-k\tau \gg 1$ which corresponds to the usual Bunch-Davies vacuum choice. This leads to

$$c_1(k) = i\frac{H}{M_p}\sqrt{\frac{\pi\epsilon}{2}}\frac{\exp\left[\frac{i\pi}{2}(\nu + 1/2)\right]}{2k}, \qquad c_2(k) = 0.$$

The solution for the metric perturbation, then, is

$$\begin{aligned}\phi^{(1)}_k(\tau) &= i\sqrt{\frac{\pi\epsilon}{2}}\frac{H}{M_p}\frac{\exp\left[\frac{i\pi}{2}(\nu + 1/2)\right]}{2k}\\ &\quad \times (-\tau)^{1/2+\eta-\epsilon}H^{(1)}_\nu(-k\tau).\end{aligned} \qquad (22)$$

It is straightforward to construct the inflaton fluctuation using (15). One may also compute the comoving curvature perturbation at first order, $\mathcal{R}^{(1)}_k = \mathcal{H}V^{(1)}_k/(a\varphi'_0)$, and verify that the solution (22) reproduces the usual inflationary prediction for the power spectrum. One may also verify that in the limit $\epsilon \to 0$ and $\eta \to 0$ the analysis of this subsection reproduces deSitter mode functions, which are discussed in appendix A.

C. Conditions for a slowly varying tachyon mass

Now we come to an important point for this paper, that if η is sufficiently small, then the tachyon is also a light field during some part of the observable period of inflation. The tachyon mass is given by $m^2_\sigma = -\lambda v^2 + g^2\varphi^2_0(t)$. If we choose the arbitrary time t_s in (14) to be when $m^2_\sigma = 0$, then $g^2\varphi^2_s = \lambda v^2$, and

$$\begin{aligned}m^2_\sigma &= -\lambda v^2\left(1 - \left(\frac{a(t)}{a_s}\right)^{-2\eta}\right) \cong -2\eta\lambda v^2 H(t - t_s)\\ &= -2\eta\lambda v^2\,N\end{aligned} \qquad (23)$$

where N is the number of e-foldings of inflation occurring after the tachyonic instability begins. At some maximum value $N = N_*$, inflation will end. If the inflaton rolls slowly enough, then the tachyon mass remains close to zero for a significant number of e-foldings. In general, we will have $-N_i \equiv N_e - N_*$ e-foldings of inflation before the spinodal time, followed by N_* e-foldings during the preheating phase.

The approximation in (23), that the tachyonic mass changes slowly enough for its time dependence to be approximated as linear, is true so long as $|2\eta N| \ll 1$. This can be rephrased using the definition of η in (13), and eliminating m_φ^2 using the COBE normalization of the inflationary power spectrum: $V/(M_p^4 \epsilon) \cong 150\pi^2 (2 \times 10^{-5})^2 = 6 \times 10^{-7}$ (see for example ref. [21]). Using $V = \frac{1}{4}\lambda v^4$ and eq. (13) for ϵ, the COBE normalization gives

$$m_\varphi^2 \cong 230\, g\lambda \, \frac{v^5}{M_p^3} \qquad (24)$$

Then with $N \sim 60$, the requirement $|2\eta N| < 1$ becomes

$$g < 10^{-5} \frac{M_p}{v} \qquad (25)$$

Interestingly, the bound (25) turns out to be a requirement that must often be satisfied for different reasons, namely the experimental limit on the spectral index of the first-order inflaton fluctuations. In terms of the slow-roll parameters, the deviation of the spectral index from unity is given by

$$n - 1 = 2\eta - 6\epsilon \cong 2\eta$$

where we have used the fact that $\epsilon \ll \eta$ in hybrid inflation (this is equivalent to the requirement that the energy density which drives inflation is dominated by $\lambda v^4/4$). The experimental constraint on the spectral index is roughly $|n-1| \lesssim 10^{-1}$. Writing η in terms of model parameters this translates into the constraint

$$g \frac{v}{M_p} \lesssim 5 \times 10^{-5} \qquad (26)$$

This is just five times weaker than the technical assumption (25).

In passing, we note that there is also a lower bound on g from the assumption that the false vacuum energy density is dominated by $\lambda v^4/4 > m_\phi^2 \varphi_0^2/2$. Using $g^2 \varphi_s^2 = \lambda v^2$ and (24), one finds

$$g > 460\, \lambda \, \frac{v^3}{M_p^3} \qquad (27)$$

D. Tachyonic Instability

To quantify the evolution of the tachyonic instability at the end of inflation, we consider the equation of motion for the tachyon field fluctuation in Fourier space,

$$\delta^{(1)}\ddot{\sigma}_k + 3H\delta^{(1)}\dot{\sigma}_k + \left[\frac{k^2}{a^2} + \left(g^2\varphi_0^2 - \lambda v^2\right)\right]\delta^{(1)}\sigma_k = 0. \qquad (28)$$

Once $\varphi_0 < \lambda^{1/2}v/g$ the tachyon effective mass parameter $\partial^2 V/\partial\sigma^2$ becomes negative and the fluctuations $\delta^{(1)}\sigma$ are amplified due to the spinodal instability. The efficient transfer of energy from the false vacuum energy $\lambda v^4/4$ to the fluctuations $\delta^{(1)}\sigma$ is refered to as *tachyonic preheating* in the literature [22]-[24].

The initial studies of tachyonic preheating focused on the flat space dynamics of $\delta^{(1)}\sigma_k$ in the instantaneous quench approximation. In this approximation the field $\delta^{(1)}\sigma$ is initially assumed to have zero mass and at $t = 0$ a negative mass squared term $-|m_\sigma^2|(\delta^{(1)}\sigma)^2/2$ is turned on. We briefly review these dynamics here following closely [22]. Initially the tachyon has the usual Minkowski space mode functions $e^{-ikt+i\vec{k}\cdot\vec{x}}/\sqrt{2k}$ [3]. Once the negative mass squared term is turned on the modes with $k = |\vec{k}| < |m_\sigma|$ grow exponentially with a dispersion

$$\left\langle (\delta^{(1)}\sigma)^2 \right\rangle = \frac{1}{4\pi^2} \int_0^{|m_\sigma|} dk\, k\, e^{2t\sqrt{|m_\sigma^2|-k^2}} \qquad (29)$$

$$= \frac{|m_\sigma^2|}{4\pi^2\alpha^2}\left(e^\alpha(\alpha-1)+1\right); \quad \alpha \equiv 2|m_\sigma|t$$

which produces a spectrum with an effective cutoff $k_{\max} = |m_\sigma|$. The tachyonic growth persists until the dispersion saturates at the value

$$\left\langle (\delta^{(1)}\sigma)^2 \right\rangle^{1/2} \cong \frac{v}{2} \qquad (30)$$

at which point the curvature of the effective potential vanishes and the tachyonic growth is replaced by oscillations about the true vacuum. This process completes within a time

$$t_s \sim \frac{1}{2|m_\sigma|} \ln\left(\frac{\pi^2}{\lambda}\right) \qquad (31)$$

which we call the spinodal time. At this point a large fraction of the vacuum energy $\lambda v^4/4$ has been converted into gradient energy of the field $\delta^{(1)}\sigma$ so that the universe is divided into domains with $\left\langle (\delta^{(1)}\sigma)^2 \right\rangle^{1/2} = \pm v$ of size $l \sim |m_\sigma|^{-1}$ and on average one still has $\langle \sigma \rangle = 0$ so that $\sigma_0(t) = 0$. These analytical argument are backed up by semi-classical lattice field theory simulations in [22, 23].

The discussion of the dynamics of tachyonic preheating above apply strictly only in Minkowski space. The dynamics of tachyonic preheating including the dynamics

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[3] Even in deSitter space the Bunch-Davies vacuum choice will ensure that this behaviour is respected on small scales $k \gg aH$ regardless of the tachyon mass during inflation. See appendix A for a review.

of the inflaton but neglecting the expansion of the universe were considered in [24]. The dynamics of tachyonic preheating including both the dynamics of the inflaton and the expansion of the universe were considered in [35] wherein the authors reach conclusions identical to those discussed above. The authors of [35] also find that the spinodal time is somewhat modified from (31) due to the background dynamics (see also [36, 37]).

Notice that we do not need to replace the average of the fluctuations $\left\langle (\delta^{(1)}\sigma)^2 \right\rangle^{1/2}$ with an effective homogeneous background $\sigma_0(\tau)$ because we work to second order in perturbation theory and the effect of these fluctuations enters into the calculation through the second order perturbed energy momentum tensor $\left\langle \delta^{(2)} T^\mu_\nu \right\rangle$. When $\left\langle \delta^{(2)} T^\mu_\nu \right\rangle$ becomes sufficiently large the backreaction will stop inflation. We take (30) as our criterion for the end of inflation. We have checked numerically that this is a somewhat more stringent constraint than demanding that the energy density in the fluctuations $\delta^{(1)}\sigma$ does not dominate over the false vacuum energy which drives inflation $\lambda v^4/4$.

In the present work, we are interested in a situation which is different from the instantaneous quench, where the instability may turn on slowly compared to the Hubble expansion, rather than suddenly. We are approximating the time dependence of the tachyon mass as being linear around the time when it vanishes, eq. (23), so the mode equation can be written in the form

$$\frac{d^2}{dN^2}\delta^{(1)}\sigma_k + 3\frac{d}{dN}\delta^{(1)}\sigma_k + \left[\hat{k}^2 e^{-2N} - cN\right]\delta^{(1)}\sigma_k = 0. \tag{32}$$

where $N = H(t - t_s)$, $\hat{k} \equiv k/H$ and, incorporating the COBE normalization as in (24),

$$c \cong 22000\, g\, M_p/v. \tag{33}$$

From eq. (25), c is limited to values

$$c \ll \left(\frac{M_p}{v}\right)^2 \tag{34}$$

for the validity of the approximation that the tachyon mass squared varies linearly with time. The quantum mechanical solution in terms of annihilation and creation operators a_k, $a_k^\dagger$ has the usual form [4]

$$\delta^{(1)}\sigma(x) = \int \frac{d^3k}{(2\pi)^{3/2}} a_k \xi_k(N) e^{ikx} + \text{h.c.} \tag{35}$$

but the mode functions ξ_k will be complicated by the time-dependence of the tachyon mass. We normalize the mode functions ξ_k according to the usual Bunch-Davies

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[4] Our conventions for fourier transforms and mode functions are discussed in detail in appendix D.

prescription which is discussed in subsection III B and also in appendix A.

Since (32) has no closed-form solution, we approximate it in two regions. First, when $\hat{k}^2 e^{-2N} > c|N|$, we ignore the mass term and use the massless solutions, $\xi_k \sim a^{-3/2} H_{3/2}^{(1)}(\hat{k}e^{-N})$. We match this onto the solution in the region where $\hat{k}^2 e^{-2N} < c|N|$, where the term $\hat{k}^2 e^{-2N}$ is ignored in the equation of motion. The transition between the two regions occurs at different times N_k for different wavelengths, given implicitly by

$$N_k = \ln\frac{\hat{k}}{\sqrt{c}} - \ln\sqrt{|N_k|} \tag{36}$$

This is a multivalued function of $x \equiv \hat{k}/\sqrt{c}$, because for $x < (2e)^{-1/2}$, the $\hat{k}^2$ term in the differential equation comes to dominate again for a short period around $N = 0$, the moment when the tachyon is massless. To deal with this, we are going to assume that the solution is still well-approximated by the massive one during this short period. This amounts to replacing the multivalued function with the single-valued one shown in figure 1. We checked this approximation in conjunction with other approximations we will make for the mode functions, as described below eq. (39).
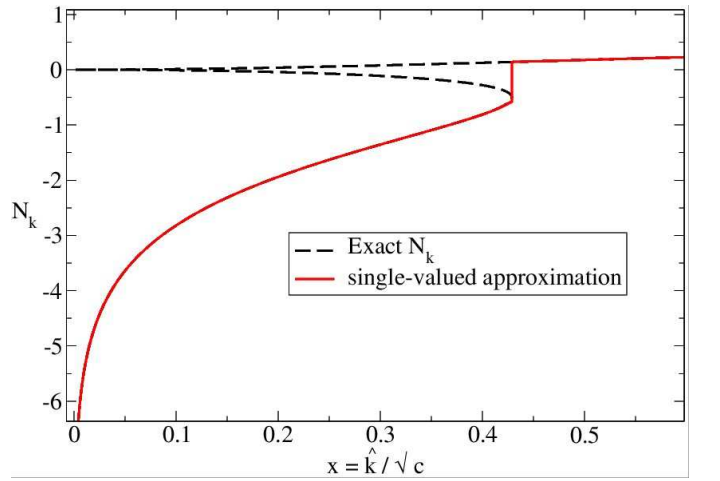

FIG. 1: Exact solution and our approximation for the function N_k in eq. (36).

In the second region, with $N > N_k$, the solutions are approximated by Airy functions, but it is more convenient to use the WKB approximation to obtain an expression in terms of elementary functions. Ignoring overall phases, in this way we obtain

$$\xi_k \cong \begin{cases} \left(2H\hat{k}^3\right)^{-1/2}\left(1 + i\hat{k}e^{-N}\right), & N < N_k \\ b_k\, e^{-\frac{3}{2}N + \frac{9}{4c}z^{3/2}}(1 + |z|)^{-1/4}, & N > N_k \end{cases} \tag{37}$$

with

$$b_k = \frac{1 - i\sqrt{c|N_k|}}{\sqrt{2H}(c|N_k|)^{3/4}} \frac{(1+|z_k|)^{1/4}}{\exp\left(\frac{9}{4c}z_k^{3/2}\right)} \qquad (38)$$

and

$$z \equiv \left(1 + \frac{4}{9}cN\right); \quad z_k \equiv \left(1 + \frac{4}{9}cN_k\right) \qquad (39)$$

In the above expressions, we have for simplicity matched the amplitudes but not derivatives of the solutions at $N = N_k$. This will not affect the estimates we make below. We used (36) to reexpress exponential dependence on N_k as power law dependence. Notice that exponent in (38) becomes purely imaginary when $\frac{4}{9}cN_k < -1$. We also replaced $|z|^{1/4} \to (1+|z|)^{1/4}$ to correct the spurious singularity at $z = 0$ where the WKB approximation breaks down. We numerically verified that this gives a good approximation to the exact Airy function solutions.

Moreover, we have checked the approximate solution by numerically integrating the mode equations, starting from the small-N region $k^2 e^{-2N} \gg c|N|$, where the massless solutions with known amplitude tell us the initial conditions, and integrating into the large-N region where the exponential growth due to the tachyonic instability becomes important. We did this for two orthogonal solutions to the mode equations, $\xi_{1,2}$, whose behavior in the small-N region is

$$\xi_1 = (2Hk^3)^{-1}e^{-N}\left(k\cos(ke^{-N}) - \sin(ke^{-N})\right)$$
$$\xi_2 = (2Hk^3)^{-1}e^{-N}\left(k\sin(ke^{-N}) + \cos(ke^{-N})\right)$$

Evolving these initial conditions to large N, the envelope of these functions, which is also the modulus of the complex solutions, is $\xi = \sqrt{\xi_1^2 + \xi_2^2}$. We compared this numerical solution to the modulus $|\xi_k|$ of our approximation (37) for a large range of c and k values. In the large N-region, $|\xi_k|$ agrees with ξ up to a numerical factor of order unity. This factor, the ratio of the actual solution to the approximation, is shown in figure (2). Because the exponential growth of the mode function is a very steep function of N, these small errors have an imperceptible effect on the exclusion plots we will present in section VI. Furthermore, we have checked which values of c and k actually give constraints in the parameter space of the hybrid inflation model below, and found that in the regions where c is large, k is exponentially small. Extrapolating the results of figure (2) indicates that the error becomes quite small as $k \to 0$. Therefore our approximations for the mode functions are quite good for the purposes of this paper.

With the above approximate solution, we are in a position to recompute the dispersion of the tachyon fluctuations in the case of a more slowly varying tachyon mass, $\langle(\delta^{(1)}\sigma)^2\rangle = (2\pi)^{-3}\int d^3k|\xi_k|^2$. Following the discussion of subsection III D, we set this equal to $v^2/4$ at the end of inflation, $N = N_*$:

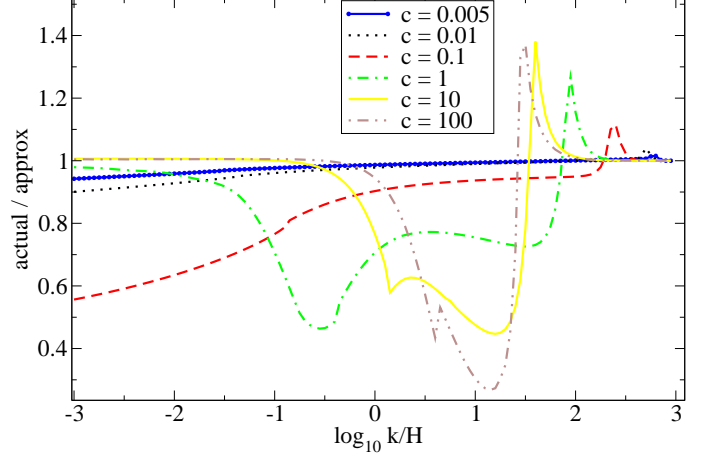$$\int \frac{d^3k}{(2\pi)^3}|\xi_k|^2\bigg|_{N=N_*} = \frac{v^2}{4} \qquad (40)$$

FIG. 2: Ratio of the exact mode functions to the approximation (37), at large times.

which implicitly determines N_* in terms of parameters of the hybrid inflation model,

$$N_* = N_*(g, \lambda, v/M_p) \qquad (41)$$

The main contribution to the integral at $N = N_*$ comes from wave numbers for which the exponentially growing solution in (37) applies. These modes satisfy $k < k_{\max} \equiv He^{N_*}\sqrt{cN_*}$. We have numerically performed the integral for a wide range of values of c and N_*. The result is displayed in figure 3, where contours of $\ln M_p^2/\lambda v^2$ are shown in the plane of N_* and $\ln c$. Recall that $c = 22000\, g\,(M_p/v)$, eq. (33).
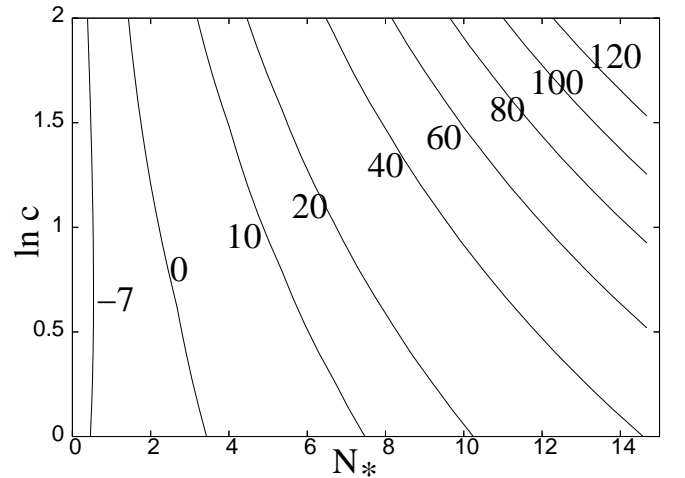

FIG. 3: Contours of $\ln M_p^2/\lambda v^2$ in the plane of N_* and $\ln c$.

IV. SECOND ORDER FLUCTUATIONS IN THE LONG WAVELENGTH APPROXIMATION

A. The Master Equation

The authors of [32] have derived a "master equation" for the second order potential $\phi^{(2)}$ which can be written as

$$\phi''^{(2)} + 2\mathcal{H}(\eta - \epsilon)\phi'^{(2)} + \left[2\mathcal{H}^2(\eta - 2\epsilon) - \partial^k\partial_k\right]\phi^{(2)} = J(\tau, \vec{x}) \qquad (42)$$

where the source terms are constructed entirely from first order quantities and can be split into inflaton and tachyon contributions

$$J(\tau, \vec{x}) = J^\sigma(\tau, \vec{x}) + J^\varphi(\tau, \vec{x}).$$

Although we have explicitly inserted the slow roll parameters, equation (42) is quite general and we have not yet assumed that η, ϵ are small (although recall that we have set $\sigma_0 = 0$). We have verified both the second order Einstein equations and the master equation presented in [32] and these results are discussed in appendix B.

Because the equation (42) is linear we can split the solutions $\phi^{(2)}$ into three parts: the solution to the homogeneous equation, the particular solution due to the inflaton souce and the particular solution due to the tachyon source. The solution to the homogeneous equation will be proportional to $\phi^{(1)}$ since the differential operator on the left hand side of (42) is identical to the operator which determines $\phi^{(1)}$, equation (16). In cosmological perturbation theory the split between background quantities and fluctuations is unambigiuous, since background quantities depend only on time while fluctuations depend on both position and time. However, the split between first order and second order fluctuations is arbitrary and the freedom to include in the solution for $\phi^{(2)}$ a contribution which is proportional to $\phi^{(1)}$ reflects this. We fix this ambiguity by including only the particular solutions for $\phi^{(2)}$ which is due to the source, $J(\tau, \vec{x})$.

During the preheating phase the tachyon fluctuations are amplified by a factor v/H and so J^σ will come to dominate. This implies that the particular solution for $\phi^{(2)}$ which is due to J^σ will come to dominate over the particular solution which is due to J^φ. Although our analysis will focus on this part of the solution, we will also consider the inflaton source in order to verify that our formalism reproduces previous results.

B. The Green Function

In this subsection we construct the Green function for the master equation so that $\phi^{(2)}$ may be determined in terms of first order quantities. As discussed previously we consider only the particular solution for $\phi^{(2)}$ due to the source J and neglect the solution to the homogeneous equation, which is equivalent to assuming that the second order fluctuations are zero before the source is turned on. During a quasi-deSitter phase the master equation can be written as

$$\partial_\tau\left[(-\tau)^{2(\epsilon-\eta)}\partial_\tau\phi_k^{(2)}\right] + (-\tau)^{2(\epsilon-\eta)}\left[\frac{2}{\tau^2}(\eta - 2\epsilon) + k^2\right]\phi_k^{(2)} = (-\tau)^{2(\epsilon-\eta)}J_k(\tau) \qquad (43)$$

where it is assumed that $\epsilon, \eta \ll 1$, and the differential operator on the left hand side of the master equation is written in a manifestly self-adjoint form. In deriving (43) we used (19) and treated the slow roll parameter as constant, which is consistent at first order in the slow roll expansion. The causal Green function for this operator is

$$G_k(\tau, \tau') = \frac{\pi}{2}\Theta(\tau - \tau')(\tau\tau')^{1/2+\eta-\epsilon} \qquad (44)$$
$$\times \left[J_\nu(-k\tau)Y_\nu(-k\tau') - J_\nu(-k\tau')Y_\nu(-k\tau)\right]$$

where the order of the Bessel functions is $\nu \cong 1/2 + 3\epsilon - \eta$ as in subsection III B.

The solution for the metric perturbation $\phi^{(2)}$ can then be written as

$$\phi_k^{(2)}(\tau) = \int_{-(1+\epsilon)/a_i H}^{0} d\tau' G_k(\tau, \tau')(-\tau')^{2(\epsilon-\eta)}J_k(\tau') \qquad (45)$$

where $a_i = a(t_i)$ is the scale factor at the some initial time, well before the tachyonic instability has set in. This solution is quite general; it applies during any slow roll phase, including during the tachyonic instability.

There are several interesting limiting cases of (44). In the long wavelength limit $k \to 0$ the Green function reduces to

$$G_k(\tau, \tau') = \Theta(\tau - \tau')(1 + 2\eta - 6\epsilon)$$
$$\left[-(-\tau)^{1+2\epsilon}(-\tau')^{2(\eta-2\epsilon)} + (-\tau')^{1+2\epsilon}(-\tau)^{2(\eta-2\epsilon)}\right].$$

In the case of pure deSitter expansion $\epsilon = \eta = 0$, but for all k, the Green function reduces to

$$G_k(\tau, \tau') = \Theta(\tau - \tau')\frac{1}{k}\sin\left[k(\tau - \tau')\right].$$

Finally, the form of the Green function in the case of $\epsilon = \eta = 0$ and $k \to 0$ may be of some interest

$$G_k(\tau, \tau') = \Theta(\tau - \tau')(\tau - \tau'). \qquad (46)$$

C. The Gauge Invariant Curvature Perturbation

The curvature perturbation is expanded to second order as

$$\zeta = \zeta^{(1)} + \frac{1}{2}\zeta^{(2)}$$

In hybrid inflation, where $\sigma_0 = 0$, the first order contribution comes entirely from the inflaton sector

$$\zeta^{(1)} = -\phi^{(1)} - \mathcal{H}\frac{\delta^{(1)}\rho}{\rho_0'}$$

where $\rho_0 = -(T_0^0)_{(0)}$, $\delta^{(1)}\rho = -\delta^{(1)}T_0^0$ are the unperturbed and first order stress tensor respectively. It is also conventional to define the comoving curvature perturbation at first order

$$\mathcal{R}^{(1)} = \phi^{(1)} + \mathcal{H}\frac{\delta^{(1)}\varphi}{\varphi_0'}$$

which, on large scales, is related to $\zeta^{(1)}$ as

$$\mathcal{R}^{(1)} + \zeta^{(1)} \cong 0.$$

The definition of the first order curvature perturbation is generally agreed upon in the literature (up to the sign of $\zeta^{(1)}$). At second order, however, there are several definitions of the curvature perturbation in the literature (see [46] for a comprehensive discussion). The definition we adopt follows [40] and generalizes the definition of Malik and Wands [41] (valid on large scales)

$$- \zeta^{(2)} = \psi^{(2)} + \frac{\mathcal{H}}{\rho_0'}\delta^{(2)}\rho - 2\frac{\mathcal{H}}{(\rho_0')^2}\delta^{(1)}\rho\,\delta^{(1)}\rho' \qquad (47)$$
$$- 2\frac{\delta^{(1)}\rho}{\rho_0'}\left(\psi'^{(1)} + 2\mathcal{H}\psi^{(1)}\right) + \frac{(\delta^{(1)}\rho)^2}{(\mathcal{H}\rho_0')^2}\left(\frac{\rho_0''}{\mathcal{H}\rho_0'} - \frac{\mathcal{H}'}{\mathcal{H}^2} - 2\right)$$

to multiple scalar fields. The definition of the curvature perturbation adopted in [32] generalizes [6] to two scalar fields. However, it has been shown that this definition applies only during inflation [45, 46] since it is conserved only in the slow roll limit.

Because $\sigma_0 = 0$ the inflationary trajectory is straight the case of hybrid inflation and $\zeta^{(1)}$ is conserved on large scales [42]-[45]. However, this is not the case at second order [46] and one expects that $\zeta^{(2)}$ will be amplified due to the tachyonic instability. This amplification of $\zeta^{(2)}$ will continue until $N = N_\star$ at which point the backreaction sets in and stops inflation. For $N > N_\star$ the inflaton is no longer dynamical since all of its energy has been converted into tachyon fluctuations. Thus for $N > N_\star$ the large scale curvature perturbation is conserved at all orders in perturbation theory since only one field (the tachyon) is dynamical and there are no non-adiabatic pressures.

In [40] the second order large scale curvature perturbation is written in terms of the first and second order

Sasaki-Mukhanov variables as

$$\zeta^{(2)} = \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\varphi_0'Q_\varphi'^{(2)} + a^2\frac{\partial V}{\partial\varphi}Q_\varphi^{(2)}\right]$$
$$+ \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\left(Q_\sigma'^{(1)}\right)^2 + a^2m_\sigma^2\left(Q_\sigma^{(1)}\right)^2\right]$$
$$+ \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\left(Q_\varphi'^{(1)}\right)^2 + a^2m_\varphi^2\left(Q_\varphi^{(1)}\right)^2\right]$$
$$+ 4(3+\epsilon-\eta)\left(\frac{3-2\epsilon}{3-\epsilon}\right)\left(\frac{\mathcal{H}}{\varphi_0'}Q_\varphi^{(1)}\right)^2$$
$$+ (-10+2\epsilon+2\eta)\left(\frac{\mathcal{H}}{\varphi_0'}Q_\varphi^{(1)}\right)^2 \qquad (48)$$

where the first order Sasaki-Mukhanov variables [5] are

$$Q_\varphi^{(1)} = \frac{\varphi_0'}{\mathcal{H}}\mathcal{R}^{(1)} \qquad (49)$$
$$Q_\sigma^{(1)} = \delta^{(1)}\sigma \qquad (50)$$

and the second order Sasaki-Mukhanov variable is

$$Q_\varphi^{(2)} = \delta^{(2)}\varphi + \frac{\varphi_0'}{\mathcal{H}}\psi^{(2)}$$
$$+ (2+2\epsilon-\eta)\frac{\varphi_0'}{\mathcal{H}}\left(\phi^{(1)}\right)^2$$
$$+ 2\frac{\varphi_0'}{\mathcal{H}^2}\phi^{(1)}\phi'^{(1)} + \frac{2}{\mathcal{H}}\phi^{(1)}\delta^{(1)}\varphi'. \qquad (51)$$

In writing (48-51) we have restricted ourselves to hybrid inflation, made use of the background equations and inserted ϵ, η (but not assuming slow roll).

The last four lines of (48) are relatively simple to evaluate in the large scale and slow roll limit. On the other hand, the first line of (48) is somewhat more complicated and its evaluation requires the second order Einstein equations. Thus we focus here on the following contribution to $\zeta^{(2)}$

$$\zeta^{(2)} \ni \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\varphi_0'Q_\varphi'^{(2)} + a^2\frac{\partial V}{\partial\varphi}Q_\varphi^{(2)}\right]. \qquad (52)$$

From the definition of $Q_\varphi^{(2)}$ it is clear that this contains a contribution of the form $\varphi_0'\delta^{(2)}\varphi' + a^2\partial V/\partial\varphi\,\delta^{(2)}\varphi$ which also appears in the second order $(0,0)$ Einstein equation (B-1). We therefore use (B-1) to eliminate $\delta^{(2)}\varphi$ from (52). We also eliminate $\psi^{(2)}$ in favour of $\phi^{(2)}$ using (B-11). The result is that (52) takes the form

$$\zeta^{(2)} \ni -\frac{\phi'^{(2)}}{\epsilon\mathcal{H}} - \left(\frac{1}{\epsilon}+1\right)\phi^{(2)} + \frac{1}{3-\epsilon}\frac{\partial^k\partial_k\phi^{(2)}}{\epsilon\mathcal{H}^2}$$
$$+ \frac{1}{\epsilon\mathcal{H}}\triangle^{-1}\gamma' + \triangle^{-1}\gamma - \frac{1}{3-\epsilon}\frac{1}{\epsilon\mathcal{H}^2}\gamma$$
$$+ \frac{1}{3-\epsilon}\frac{\Upsilon_1}{\epsilon\mathcal{H}^2} + \frac{1}{3-\epsilon}\left(\frac{\Upsilon_4'}{\varphi_0'} + \frac{a^2}{(\varphi_0')^2}\frac{\partial V}{\partial\varphi}\Upsilon_4\right)(53)$$

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[5] Notice that $Q_\varphi^{(1)}$ is related to the variable $V^{(1)}$ discussed in subsection III B by $V^{(1)} = a\,Q_\varphi^{(1)}$.

where the quantities Υ_1, γ are constructed entirely from first order fluctuations and are defined explicitly in appendix B in equations (B-2, B-3, B-5, B-6, B-8, B-9). The quantity Υ_4 is also contructed from first order fluctuations and is defined as the last two lines of (51). That is,

$$\begin{aligned}\Upsilon_4 &= (2 + 2\epsilon - \eta)\frac{\varphi_0'}{\mathcal{H}}\left(\phi^{(1)}\right)^2 \\ &+ 2\frac{\varphi_0'}{\mathcal{H}^2}\phi^{(1)}\phi'^{(1)} + \frac{2}{\mathcal{H}}\phi^{(1)}\delta^{(1)}\varphi'.\end{aligned} \tag{54}$$

From (53) we see that the dependence of $\zeta^{(2)}$ on the second order fluctuations comes from the combination on the first line of (53) which can be computed in terms of the source using the Green function (44).

We split $\zeta^{(2)}$ into contributions coming from the inflaton and the tachyon as

$$\zeta^{(2)} = \zeta_\varphi^{(2)} + \zeta_\sigma^{(2)}$$

and study each piece separately. This splitting is different from the one discussed in [40], where the curvature perturbation is defined for each fluid in such a way that the total curvature perturbation is a weighted sum of the individual contributions. Instead, we simply divide $\zeta^{(2)}$ into terms which depend respectively on the tachyon and inflaton fluctuations, $\delta^{(1)}\sigma$ and $\delta^{(1)}\varphi$, $\phi^{(1)}$, which is only possible because $\sigma_0 = 0$.

The tachyon part of the curvature perturbation $\zeta_\sigma^{(2)}$ gets contributions from the first and second line of (48), both explicitly through $Q_\sigma^{(1)}$ and implicitly through $\delta^{(2)}\varphi, \psi^{(2)}$. The inflaton part of the curvature perturbation $\zeta_\varphi^{(2)}$ contains contributions from the last three lines of (48) as well as from the first line of (48), both implicitly through $\delta^{(2)}\varphi$, $\psi^{(2)}$ and explicitly through the definition of $Q_\varphi^{(2)}$.

The inflaton part of the curvature perturvation, $\zeta_\varphi^{(2)}$ coincides with the $\zeta^{(2)}$ of single field inflation and has been derived previously [7, 47, 48]. We have considered the construction of $\zeta_\varphi^{(2)}$ using our formalism and these results are presented in appendix C.

D. The Tachyon Curvature Perturbation

We now consider the tachyon contribution to the second order curvature perturbation $\zeta_\sigma^{(2)}$. It is sourced by J^σ which, in position space, takes the form (see equation (B-16))

$$\begin{aligned}J^\sigma(\tau, \vec{x}) &= a^2\kappa^2 m_\sigma^2\left(\delta^{(1)}\sigma\right)^2 - 2\kappa^2\left(\delta^{(1)}\sigma'\right)^2 \\ &+ 2\kappa^2\mathcal{H}(1 + \eta - \epsilon)\triangle^{-1}\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\ &+ 4\kappa^2\triangle^{-1}\partial_\tau\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\ &- \mathcal{H}(1 + 2\epsilon - 2\eta)\triangle^{-1}\gamma_\sigma' + \triangle^{-1}\gamma_\sigma''.\end{aligned} \tag{55}$$

The quantity γ_σ can be written in the form (see equation (66) of [32], or equuivalently (B-15))

$$\begin{aligned}\gamma_\sigma &= -\kappa^2\triangle^{-1}\Big[3\partial_i\left(\partial^k\partial_k\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right) \\ &+ \frac{1}{2}\partial^k\partial_k\left(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right)\Big] \\ &= -3\kappa^2\triangle^{-1}\partial_i\left(\partial^k\partial_k\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right) \\ &- \frac{\kappa^2}{2}\left(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right).\end{aligned}$$

Notice that the terms in the first line of the (55) are local, the terms in the second and third line are non-local (containing an inverse laplacian $\triangle^{-1}$) and the fourth line contains terms which are both local and doubly non-local (containing $\triangle^{-2}$). The Fourier transforms of the source terms are computed in appendix D wherein we also discuss our conventions for the inverse laplacian operators.

In the following, we will need the Fourier transform of terms like $\triangle^{-1}\gamma_\sigma$,

$$\begin{aligned}\mathcal{F}\left[\triangle^{-1}\gamma_\sigma\right] &= -3\frac{\kappa^2}{k^4}\int\frac{d^3k'}{(2\pi)^{3/2}}k'^2 k \cdot (k - k')\delta^{(1)}\tilde{\sigma}_{k'}\delta^{(1)}\tilde{\sigma}_{k-k'} \\ &- \frac{\kappa^2}{2k^2}\int\frac{d^3k'}{(2\pi)^{3/2}}k' \cdot (k - k')\delta^{(1)}\tilde{\sigma}_{k'}\delta^{(1)}\tilde{\sigma}_{k-k'}\end{aligned}$$

This expression is operator-valued and can be written in terms of annihilation/creation operators and mode functions as $\delta^{(1)}\tilde{\sigma}_k = a_k\xi_k(t) + a_{-k}^\dagger\xi_{-k}(t)$ (see appendix D for more details). In the Fourier transformed expression for $\triangle^{-1}\gamma_\sigma$, the scale dependence of the mode functions $\delta^{(1)}\sigma_k$ is integrated over so that $\triangle^{-1}\gamma_\sigma$ gets contributions from the tachyon fluctuations on all scales. The large scale limit of terms like $\triangle^{-1}\gamma_\sigma$ is not transparent and therefore we do not neglect any terms in the tachyon source which contain inverse Laplacians.

We now compute $\zeta_\sigma^{(2)}$. The tachyon contribution to $\zeta^{(2)}$ comes about entirely through the implicit dependence of $\delta^{(2)}\varphi$ and $\psi^{(2)}$ on $\delta^{(1)}\sigma$ in the first line of (48) and the explicit dependence on the second line of (48). Our focus is on the leading order contribution to $\zeta^{(2)}$ in the slow roll and large scale limit. If we work only to leading order in the slow roll parameters it is sufficient to use the Green function (44) in the limit $\epsilon = \eta = 0$ and keep only the terms in the tachyon source (55) which are not slow-roll suppressed. However, to consistently compute $\zeta_\sigma^{(2)}$ we must keep the next-to-leading order terms in the small $(k/\mathcal{H})^2$ expansion of the Green function. To see this notice that powers of k^2 cancel inverse Laplacians in the source:

$$k^2 J_k^\sigma = -\gamma_{\sigma,k}'' + \mathcal{H}\gamma_{\sigma,k}' + \cdots$$

where $\cdots$ denotes gradient terms which are small on large scales. In appendix B it is shown that γ_σ can be written

on large scales as (B-14)

$$\gamma_\sigma \cong \frac{3\kappa^2}{2}\left[\left(\delta^{(1)}\sigma'\right)^2 - a^2 m_\sigma^2 \left(\delta^{(1)}\sigma\right)^2\right]$$
$$- 3\kappa^2 \triangle^{-1}\partial_\tau\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right)$$
$$- 6\mathcal{H}\kappa^2 \triangle^{-1}\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right).$$

Thus $k^2 J_k^\sigma/\mathcal{H}^2$ contains terms which are of the same form as those which appear in J_k^σ (55) and hence these terms must be included to consistently study $\zeta_\sigma^{(2)}$ on large scales.

The curvature perturbation $\zeta^{(2)}$ depends on $\phi^{(2)}$ through the combination $-\phi'^{(2)}/\epsilon\mathcal{H} - \phi^{(2)}/\epsilon + \partial^k\partial_k\phi^{(2)}/3\epsilon\mathcal{H}^2$ at leading order in slow roll parameters (53). Using the Green function for $\epsilon = \eta = 0$ expanded up to order k^2 we find that

$$- \frac{\phi_k'^{(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} =$$
$$\frac{1}{\epsilon}\int_{\tau_i}^0 d\tau'\Theta(\tau - \tau')J_k^\sigma(\tau')$$
$$\times \left[\tau' + \left(-\frac{1}{6}\tau'^3 + \frac{5}{6}\tau^2\tau' - \frac{2}{3}\tau^3\right)k^2\right]. \quad (56)$$

Integrating (56) by parts[6] and plugging the result into (48) and (53) we find that on large scales

$$\zeta_\sigma^{(2)} \cong \frac{\kappa^2}{\epsilon}\int_{-1/a_i H}^{\tau} d\tau'\left[\frac{\left(\delta^{(1)}\sigma'\right)^2}{\mathcal{H}(\tau')}\right.$$
$$\left. - \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}\left(\left(\delta^{(1)}\sigma'\right)^2 - a^2 m_\sigma^2 \left(\delta^{(1)}\sigma\right)^2\right)\right] \quad (57)$$

where the tachyon fluctuations $\delta^{(1)}\sigma$ are functions of the integration variable τ'. The corrections to (57) are either subleading in the slow roll expansion or are total gradients which can be neglected on large scales. Equation (57) is the main result of this section. The interested reader may find a more detailed discussion of our calculation of $\zeta_\sigma^{(2)}$ in appendix E.

It is interesting to contrast the simplicity of this result with other previous computations of ostensibly the same quantity [32]. Part of the difference is due to our different definition of the second order curvature perturbation, but one must also take considerable care in keeping all terms which are of the same order in slow roll parameters and

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[6] This integration by parts will give rise to terms which are evaluated between the initial and final time of the form $[\cdots]_{\tau'=-1/a_i H}^{\tau}$. Since we are interested in the preheating phase during which the fluctuations are amplified exponentially we can safely drop the contribution at $\tau' = -1/a_i H$ relative to the contribution at $\tau' = \tau$.

powers of k^2 in order to obtain the delicate cancellations that collapsed the sum of many terms down to this compact form. Especially notable is the absence of nonlocal terms (containing inverse Laplacians) in the final result, which were quite prevalent in the intermediate steps. It is an important consistency check that such terms disappear in the end, since they do not respect causality. For example, a term of the form $\triangle^{-1}\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma$ gives rise to an acausal response to a source which is localized in time and space. Consider a source of the form $\delta^{(1)}\sigma \sim e^{-\vec{x}^2/a^2}$ which is turned on at some instant in time. Then $\triangle^{-1}\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma \sim 1/|\vec{x}|$ instantaneously, at large distances, instead of being exponentially small. This would be a clear violation of causality and is physically inadmissible.

To clarify, we have shown that at leading order in slow roll parameters the nonlocal contributions to $\zeta_\sigma^{(2)}$ cancel. We have not checked that such terms cancel at higher order in the slow roll expansion, though we believe that they do. This cancellation was not observed in previous studies because the subleading (in k^2) corrections to the Green function were not included and thus the large scale expansion was inconsistent.

A comment is in order concerning the long wavelength approximation which we have used in deriving (57). In writing the expression for $k^2 J_k^\sigma$ we have dropped terms which are total gradients even though such terms will be integrated over time in computing $\zeta_\sigma^{(2)}$, due to the time integral in (45). Strictly speaking, one should only apply the large scale limit $k\tau \ll 1$ *after* the time integral has been performed. The reason for this is that the time integral extends from when the modes are well within the horizon, where they oscillate, to when the modes are outside the horizon, including horizon crossing. Mode-mode coupling near the epoch of horizon crossing can contribute momentum-dependent terms which are not suppressed on large scales [7, 50]. However in our application, we are justified in dropping the contributions which are generated near horizon crossing since they are exponentially suppressed compared those which are produced during the preheating phase.

E. Time-integrated tachyon perturbation

We can make our main result (57) more explicit by substituting in the solutions (35,37). The result is simplified by taking the Fourier transform of $\zeta^{(2)}$ and evaluating it at vanishing external wave number, and at the final time corresponding to the end of inflation, $N = N_*$:

$$\zeta_{k=0}^{(2)}(N_*) = \frac{\kappa^2}{\epsilon}\int \frac{d^3 p}{(2\pi)^{3/2}}\left(a_p\, b_p\, a_p^\dagger\, b_{-p} + \text{perms}\right)$$
$$\times \int_{\max(N_p,N_i)}^{N_*} f(c, N, N_*)\, dN \quad (58)$$

where N_i denotes the value of N at the beginning of inflation, $f(c, N, N_*)$ is given by

$$f(c, N, N_*) = e^{-3N + \frac{9}{2c} z^{3/2}} \left(1 + |z|\right)^{-1/2} \times$$

$$\left[\frac{9}{4} \left(1 - e^{3(N - N_*)}\right) \left| \sqrt{z} - 1 - \frac{2c \, \text{sign}(z)}{27(1 + |z|)} \right|^2 - \right.$$

$$\left. cN e^{3(N - N_*)} \right] \qquad (59)$$

where $z \equiv (1 + \frac{4}{9}cN)$, and "perms" in (58) indicates the three other combinations of $a_p \, b_p$ and $a_p^\dagger \, b_{-p}$.

For illustration, we show the behavior of the function $f(c, N, N_*)$ for sample parameter values $N_* = 22.5$ and $\ln c = 1$. Figure 4 plots $(\text{sign}(f) \ln(1 + |f|)$ as a function of N. The function is exponentially peaked at the initial value $N = N_i$, and at the final value $N = N_*$. Moreover it always becomes negative at N_* because the negative mass squared term in (59) comes to dominate. Although it is not obvious in the figure, the negative value is orders of magnitude larger than the positive maximum just preceding it, so the negative extremum dominates in the integral in (59). Whether the extremum at N_* or N_i dominates overall depends on how $|3N_i|$ compares to $9/(2c)z^{3/2}$ evaluated at N_*.

Because of the exponential growth of f at its extrema, it is a very good approximation to the integral to write $f = e^g$ and expand $g = g_m + g'_m(N - N_m)$ in the vicinity of the maximum value, whether it is at N_i or N_*. Since the integral is so strongly peaked near the extremum, there is an exponentially small error in extending the range of integration to the half-line. In this way one obtains

$$\int dN \, f \cong \frac{e^{g_m}}{|g'_m|} \qquad (60)$$

We will use this approximation below to numerically evaluate the integral.

V. BISPECTRUM AND SPECTRUM OF SECOND ORDER METRIC PERTURBATION

Here we calculate the leading contribution to the three-point function (bispectrum) of the second-order curvature perturbation due to the tachyon,

$$\left\langle \zeta^{(2)}_{k_1} \zeta^{(2)}_{k_2} \zeta^{(2)}_{k_3} \right\rangle \equiv (2\pi)^{-3/2} \, \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \, B(\vec{k}_1, \vec{k}_2, \vec{k}_3) \qquad (61)$$

It is understood that only the connected part of the correlation function is computed, which is equivalent to subtracting the expectation value of $\zeta^{(2)}$ from the quantum operator. This three-point function is straightforward to compute, using the free-field two-point functions

$$\left\langle \delta^{(1)} \tilde{\sigma}_{p_i} \delta^{(1)} \tilde{\sigma}_{q_i} \right\rangle = \xi_{p_i} \xi_{q_i} \, \delta^{(3)}(p_i + q_i) \qquad (62)$$
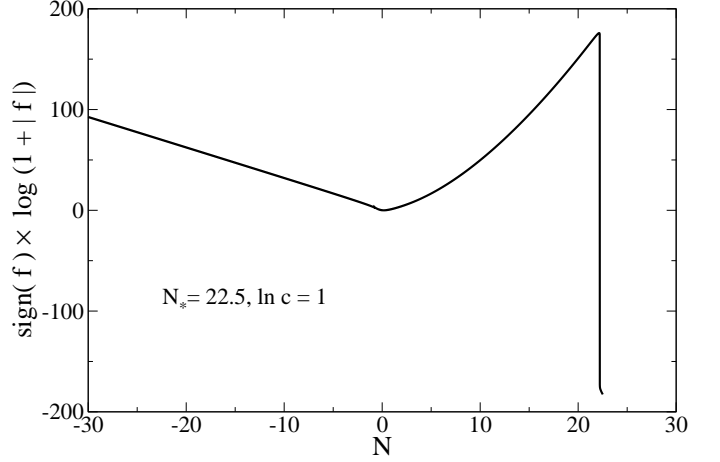

FIG. 4: $(\text{sign}(f) \ln(1 + |f|)$ versus N, showing behavior of the the function f defined in (59), for $\ln c = 1$, $N_* = 22.5$

Carrying out the contractions of pairs of fields which contribute to the connected part of the bispectrum, one finds eight terms, which in the limit of vanishing external wave-numbers, are all equal. The result is

$$B = 8 \frac{\kappa^6}{\epsilon^3} \int \frac{d^3 p}{(2\pi)^3} |b_p|^6 \left[\int_{\max(N_p, N_i)}^{N_*} dN \, f(c, N, N_*) \right]^3 \qquad (63)$$

The integrand of the p-integral is exponentially strongly peaked, either near $\ln(p/\sqrt{c}H) \cong N_i$ if f has its global maximum near $N = N_i$, or else near $\ln(p/\sqrt{c}H) \cong 0$ if f is dominated by its behavior near N_*. The logarithm of the integrand for a typical case is shown in figure 5. If the cusp-like peak is dominating, we can use the same approximation as in (60) to evaluate the p integral, on both sides of the maximum. If the other local maximum at larger values of p is the global maximum, as sometimes happens, then we should treat the integral as a gaussian, since the derivative of the integrand vanishes at the maximum. In this case we similarly write the integrand of the p integral as $f = e^g$ and expand $g = g_m + \frac{1}{2}g''_m(p - p_m)$, where $g''_m < 0$; then (60) is replaced by

$$\int dN \, f \cong \sqrt{\frac{2\pi}{|g''_m|}} e^{g_m} \qquad (64)$$

We have carried out the evaluations numerically over a range of values of $\ln c$ and N_*. In figure (6) we plot contours of $\ln |\tilde{B}|$, where $\tilde{B}$ is defined by

$$B = \frac{e^{-6N_i}}{2\pi^2} \left(\frac{\kappa^2}{c\epsilon} \right)^3 \tilde{B} \qquad (65)$$

Although we evaluated B at vanishing external wave numbers, $k_i = 0$, the result is equivalent to evaluating B at very small wave numbers closer to horizon crossing,
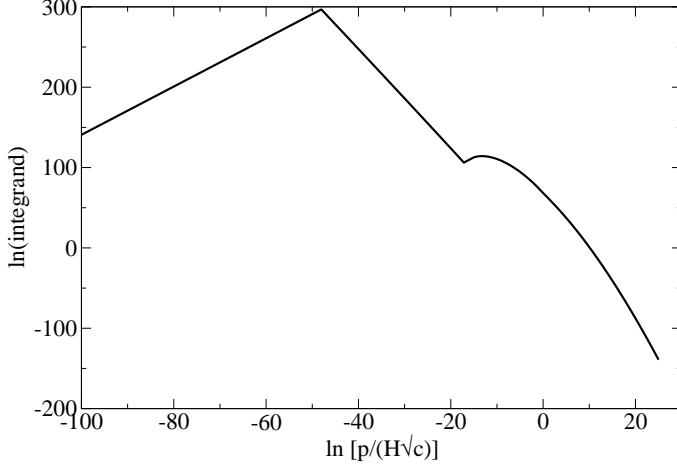
FIG. 5: Log of the integrand of the p integral in eq. (63), for a case where the maximum occurs near $\ln(p/\sqrt{c}H) = N_i$.
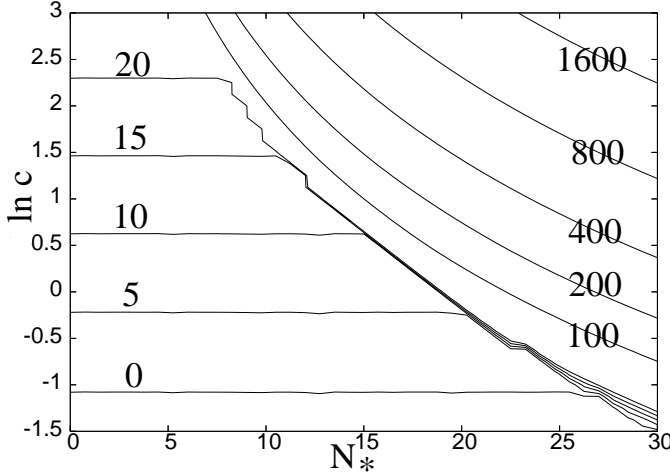

FIG. 6: Contours of $\ln|\tilde{B}|$, defined in (65), in the plane of N_* and $\ln c$. $N_i = -30$ in this example.

$k_i \cong He^{N_i}$. This is not immediately obvious, but one can see that at very small p, the p integral in (63) behaves like $\int d^3p/p^9$, which in the presence of nonvanishing external wave numbers would be $\int d^3p \, p^{-3}(p-k)^{-6} \sim k^{-6}$. (This claim is explicitly verified in appendix F.) However, (63) does not diverge as $k \to 0$; the p integral is cut off near He^{N_i} because of the lower limit $\max(N_i, N_p)$ in the integral over N. If the lower limit were simply N_p instead of $\max(N_i, N_p)$, the value of the N integral near its lower limit would go like $e^{3N_p} \sim 1/p^3$ instead of e^{N_i}. Thus the factor of e^{6N_i} in (65) is a reflection of the fact that the bispectrum (at least the part of it which is due to the low-p part of the integral) is scale-invariant, $B \sim 1/k^6$, for $k > He^{N_i}$, but the infrared divergence is cut off for $k < He^{N_i}$, and the spectrum remains flat as $k \to 0$.

A consequence of the above discussion is that the bis-

pectrum is scale-invariant only in the part of parameter space where the low-N part of the N integral dominates, namely the lower left-hand region of fig. (6). The upper right-hand region of fig. (6) gets its dominant contributions from the large-p part of the p integral, which is not infrared sensitive. The bispectrum really is flat as a function of k, for small k, in this region. Furthermore, B is negative in this region, whereas it is positive in the other region. This means there is a very narrow, fine-tuned curve along which B vanishes, seen as the borderline between the two different behaviors. This is a region which will always be allowed as long as no nongaussianity is observed in the CMB, but since it is a set of measure zero in the parameter space, we will not take it into account in deriving limits below.

The boundary between the two regions in fig. (6) is described analytically by the relation

$$3|N_i| = \frac{9}{2c}z_*^{3/2} \qquad (66)$$

with $z_* = 1 + \frac{4}{9}cN_*$. This where the two terms in the exponent of f in (59), which determinant the dominant behavior of f, just balance. In the region to the right, where $\frac{9}{2c}z_*^{3/2} > 3|N_i|$, which is also where B is independent of k_i, B is also independent of N_i. In the region to the left, B depends principally on N_i through the factor e^{-6N_i} in (65). However there is some mild extra dependence on N_i, as shown in figure (7).
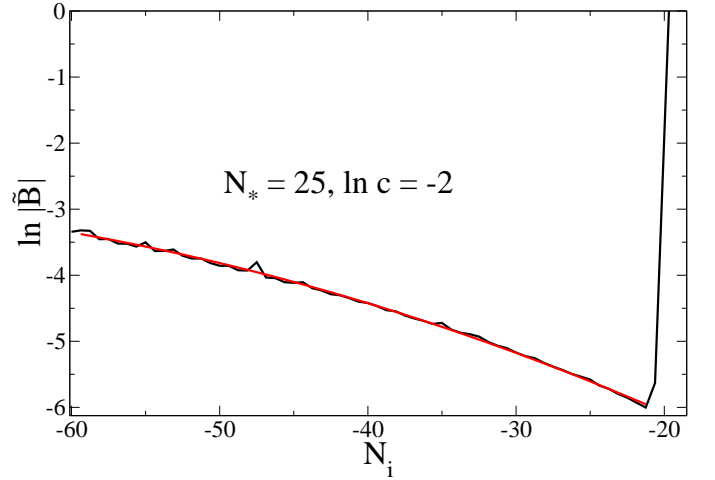

FIG. 7: Dependence of $\ln|\tilde{B}|$ on N_i for $N_* = 25$ and $\ln c = -2$. The smooth curve is the numerical fit $\ln|\tilde{B}| = -8.3 - 0.13N_i - 0.0007N_i^2$.

To make contact with experimental constraints, we want to compare the predicted bispectrum with that of single-field inflation, where nongaussianity is conventionally expressed via a nonlinearity parameter f_{NL}, defined through

$$B(k_i) = -\frac{6}{5}f_{NL}\left(P_\phi(k_1)P_\phi(k_2) + \text{ permutations}\right) \quad (67)$$

where $P_\phi(k)$ is the usual inflationary power spectrum, $P_\phi(k)^{1/2} \sim 10^{-5} (2\pi)/k^{3/2}$. If we assume that all k_i are the same, $k_i \cong k$, then

$$f_{NL} = -\frac{5}{18}(2\pi)^{-4}10^{20} B(k) k^6 \qquad (68)$$

We can thus convert our predicted bispectrum into an effective f_{NL}, for which the present experimental constraint is roughly $|f_{NL}| < 100$. To get the strongest constraint, k should be evaluated at scales near horizon crossing in the scale-invariant region of B, and at the smallest relevant scales in the region where B is k-independent. Since $B(k)$ is evaluated at the end of inflation, the horizon-crossing scale is $k \sim He^{-N_e}$. On the other hand, current experimental constraints on nongaussianity involve temperature multipoles of the CMB going up to $\ell = 265$ [51], which represents scales roughly e^5 times smaller than horizon-crossing. So we should use $k = e^{-N_e+5}H$ in the region where B is constant at low k. It turns out that this is the region of parameter space where the experimental limit is saturated, so it is the relevant case.

Using (65) in (68) and recalling that $N_e = |N_i| + N_*$ we find that

$$f_{NL} = -e^{17.5-6N_*}c^{-3}\tilde{B} \qquad (69)$$

where we used $H^2 = V/(3M_p^2)$ as well as the COBE normalization $V/(\epsilon M_p^4) = 6 \times 10^{-7}$. The contours of $\tilde{B}$ can thereby be converted into contours of f_{NL}, and demanding that $|f_{NL}| < 100$ gives a constraint in the parameter space c, N_*, N_i. The limiting curves are shown for different values of N_i in figure 8. These curves are quite insensitive to the actual value assumed for f_{NL}, because the effect turns on exponentially fast. The curves for $f_{NL} = 1$ are visually hard to distinguish from those shown (for $f_{NL} = 100$).
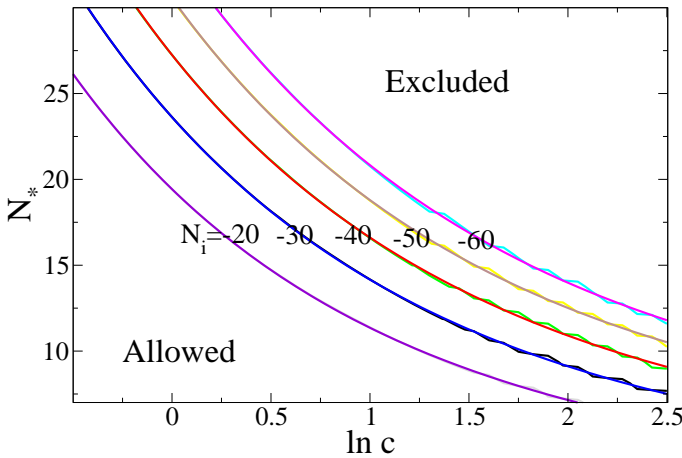

FIG. 8: Boundaries of the regions excluded by the nongaussianity constraint $|f_{NL}| < 100$, in the plane of $\ln c$ and N_*, for different values of N_i.

A. Constraint from tachyon contribution to spectrum

The analogous calculation can also be carried out for the tachyon contribution to the two-point function of the curvature perturbation. We show that it gives a stronger constraint than does nongaussianity.

Closely following the preceding calculation, it is straightforward to show that, in the limit of vanishing external wave numbers,

$$\left\langle \zeta_{k_1}^{(2)} \zeta_{k_2}^{(2)} \right\rangle_{con} = \delta(\vec{k}_1 + \vec{k}_2) \times$$
$$2\frac{\kappa^4}{\epsilon^2} \int \frac{d^3p}{(2\pi)^3}|b_p|^4 \left[\int_{N_p}^{N_*} dN\, f(c, N, N_*)\right]^2$$
$$\equiv \delta(\vec{k}_1 + \vec{k}_2)\, S(k_i) \qquad (70)$$

In analogy to (65), we define[7]

$$S = \frac{e^{-3N_i}}{4\pi^2} \frac{H}{c^{3/2}} \left(\frac{\kappa^2}{\epsilon}\right)^2 \tilde{S} \qquad (71)$$

and we display contours of $\tilde{S}$ in fig. 9. Analogously to the bispectrum, S is approximately scale-invariant ($\sim 1/k_i^3$) in the left-hand region, but independent of k in the right-hand region.
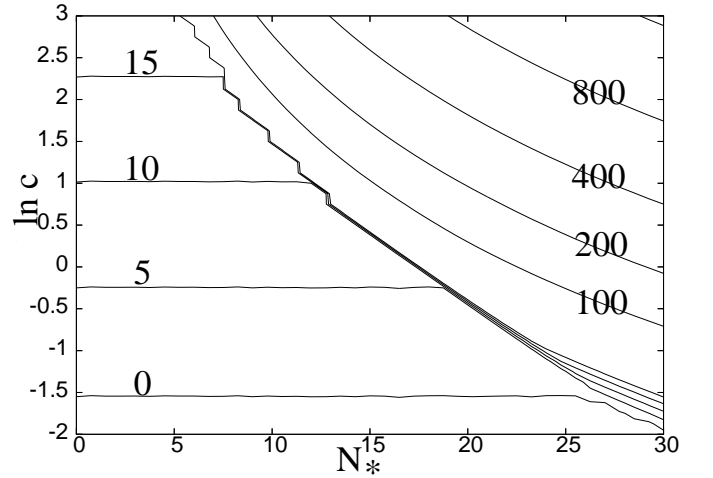

FIG. 9: Contours of $\ln|\tilde{S}|$ in the plane of N_* and $\ln c$, at $N_i = -30$.

The COBE normalization of the power spectrum implies that

$$\left\langle \zeta_{k_1}^{(2)} \zeta_{k_2}^{(2)} \right\rangle_{con} \leq \frac{2\pi^2}{k^3} \delta(\vec{k}_1 + \vec{k}_2)\, \mathcal{P}_\zeta \qquad (72)$$

---

[7] The powers of H and c can be understood as follows: $b_p \sim H^{-1/2}c^{-3/4}$, whereas $p \sim \sqrt{c}H$.

with $\mathcal{P}_\zeta^{1/2} \cong 10^{-5}$. Combining this with (70) and (71), we get an upper bound on the quantity

$$f_L \equiv \frac{10^{20}}{(2\pi)^3} \frac{H}{c^{3/2}} \left(\frac{\kappa^2}{\epsilon}\right)^2 \tilde{S}\, e^{-3N_i}\, k^3 < 1 \qquad (73)$$

where the strongest constraint is obtained by taking the largest k values which are measured by the CMB. We will conservatively take this to be $k = e^6 H e^{-N_e}$. Notice that the tachyon fluctuations have a spectral index of $n = 4$ (defined by $k^3 S \sim k^{n-1}$) which is consistent with the Traschen integral constraints [54]. Following the same steps as we did for the bispectrum, this can be rewritten as

$$f_L = e^{30-3N_*}\, c^{-3/2}\, \tilde{S} \left\{ \begin{array}{ll} 1, & \frac{9}{2c}z_*^{3/2} > 3|N_i| \\ e^{-18}, & \frac{9}{2c}z_*^{3/2} < 3|N_i| \end{array} \right\} < 1 \qquad (74)$$
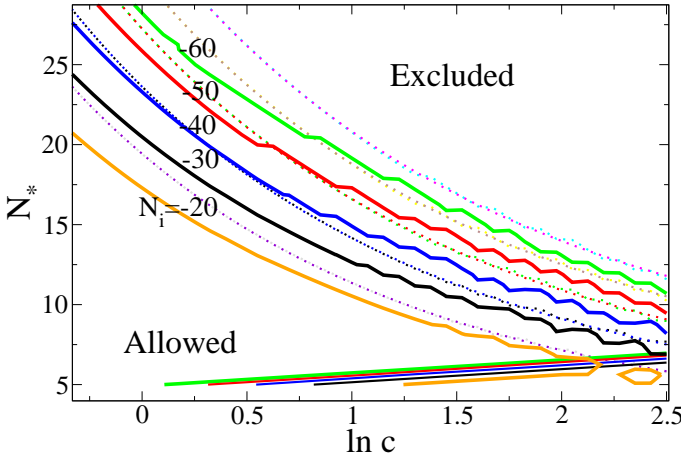

FIG. 10: Boundaries of the regions excluded by the spectral constraint $|f_L| < 1$, in the plane of $\ln c$ and N_*, for different values of N_i. The corresponding constraints from f_{NL} (fig. 8) are shown as dotted lines.

One might consider being more conservative and imposing, say, $|f_L| < 0.01$ rather than $|f_L| < 1$, as we have done. Our exclusion plots are actually quite insensitive to the value assumed for f_L, as was the case with f_{NL}, since the effect turns on exponentially fast.

VI. CONSTRAINTS ON HYBRID INFLATION MODEL PARAMETERS

In the previous section we computed the effective non-linearity parameter f_{NL} which characterizes the size of nongaussian fluctuations produced by tachyonic pre-heating in the hybrid inflation model under consideration. Experimentally, it is currently constrained as $|f_{NL}| \lesssim 100$ [51], which is expected to improve to the level of $|f_{NL}| < 5$ from future experiments, like PLANCK

[9, 52]. Similarly, we defined a parameter f_L which characterizes the size of the second-order tachyonic contribution to the spectrum relative to the experimental value, with the constraint $|f_L| < 1$. These constraints were most easily determined in terms of derived parameters c and N_*, but here we want to recast them in terms of the fundamental parameters of the hybrid inflation model, the dimensionless couplings g and λ, and the VEV of the tachyon field, expressed in the dimensionless combination v/M_p. We do not treat m_φ^2 as an independent parameter, since we used the COBE normalization to eliminate it in (24).

The only obstacle to working directly in the model parameter space is the implicit dependence of N_* on (λ, g, v). We have numerically inverted the relation depicted in fig. (3) to determine $N_*(\lambda, g, v)$. It then becomes straightforward to scan the model parameter space, recomputing f_L and f_{NL}. We display the constraints in the plane of $\log_{10} g$ and $\log_{10} \lambda$, for a range of values of $\log_{10}(v/M_p)$. In performing this scan, one must keep track of whether various assumptions are respected. These include eqs. (25) and (27), respectively corresponding to the assumption that m_σ^2 varies linearly with the number of e-foldings, and that the vacuum energy density is dominated by $\lambda v^4/4$ during inflation. In addition to these, we implement two others: we assume that (1) the reheat temperature exceeds 100 GeV, so that baryogenesis can occur at least during the electroweak phase transition; and (2) we limit ourselves to considering values $c \geq 10^{-4}$, which entails a lower limit on the coupling g depending on v, eq. (40). This is a numerical limitation, due to the difficulty of evaluating the integral (40) for small c. However, analytic calculations can be done in this region, which we have carried out in Appendix F, as we describe below.

In the region where the condition (25), $gv/M_p < 10^{-5}$, is not satisfied our analysis breaks down because the tachyon mass m_σ^2 varies exponentially rather than linearly with the number of e-foldings. However, we do not expect that any significant nongaussianity will be generated in the region where c is too large because in this case we will have had $m_\sigma^2 > H^2$ for a significant number of e-foldings before the instability sets in and the fluctuations of the tachyon will have a large exponential suppression. However, to be conservative in our analysis we consider the region where (25) is not satisfied as allowed by the experimental constraints on f_L, f_{NL}. This is a rather small sliver of parameter space, since the spectral index contstraint (26) requires that $gv/M_p < 5 \times 10^{-5}$.

Once a set of values for λ, g and v/M_p are chosen and N_* has been calculated, one must still determine N_i in order to calculate the parameters f_L and f_{NL}. We do so by first computing the total number of e-foldings using the standard result

$$N_e = 62 - \ln\left(\frac{10^{16}\text{ GeV}}{V^{1/4}}\right) - \frac{1}{3}\ln\left(\frac{V^{1/4}}{\rho_{\text{r.h.}}^{1/4}}\right) \qquad (75)$$

where $V \cong \lambda v^4/4$ is the energy density during inflation,

and $\rho_{\mathrm{r.h.}}$ is the energy density at reheating. In the following, we will ignore the gravitino bound $\rho_{\mathrm{r.h.}}^{1/4} \lesssim 10^{10}$ GeV and assume instant reheating, $\rho_{\mathrm{r.h.}} = V$. The value of N_i then follows from $N_i = N_* - N_e$. We have checked that incorporation of the gravitino bound does not create a noticeable change in the excluded regions.

The resulting constraints are illustrated for the case where $v/M_p = 10^{-3}$ in figure 11. The unshaded region is excluded by our calculation of f_L and f_{NL}. The shaded region on the right falls outside our approximation (25) or the spectral index constraint (26), that on top fails to satisfy (27), and the region on the bottom has too low a reheat temperature. The region on the left has $c < 10^{-4}$ and we were not able to treat it using the same numerical methods, but we analyzed this region analytically in Appendix F. The tachyon mass is small throughout inflation in this region, and gives rise to a scale-invariant spectrum of nongaussian fluctuations, unlike the excluded region shown in figure 11, which corresponds to a scale-noninvariant contribution, with spectral index $n = 4$. Nevertheless, the magnitude of nongaussianity in this region labeled, "also excluded," exceeds the experimental bounds.

FIG. 12: Closeup of the borderline region in figure 11, showing that the spectral distortion ($f_L < 1$, dashed line) provides the stronger constraint, relative to nongaussianity ($f_{NL} < 100$, solid line).
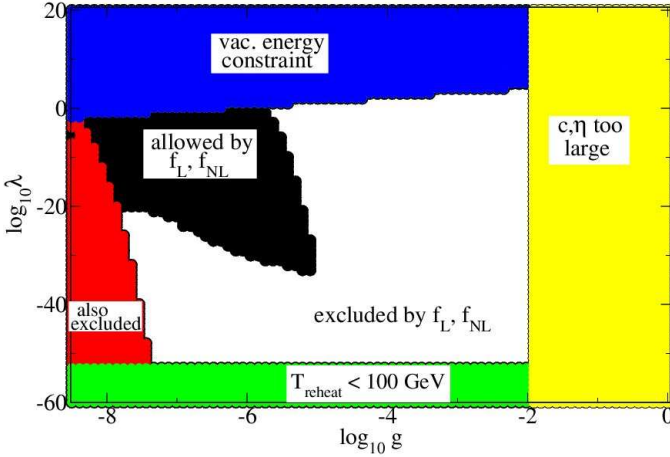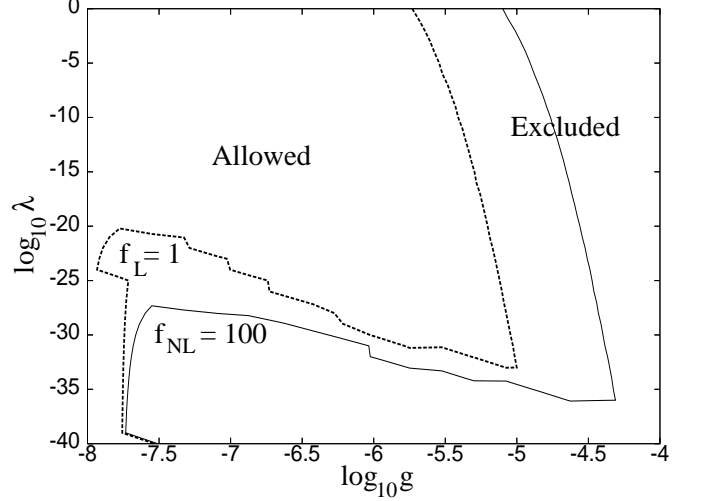

FIG. 11: Unshaded region in the plane of $\log_{10} g$ and $\log_{10} \lambda$ is excluded by the tachyon contribution to the second order curvature perturbation, for $v/M_p = 10^{-3}$. Leftmost region is also excluded by a separate analysis (Appendix F).

As was indicated in the previous section, we get a stronger constraint from the spectrum, whose distortions are parametrized by f_L (73), than from nongaussianity, parameterized by f_{NL}. This is shown in figure 12, which plots the contours for the constraints $f_L < 1$ and $f_{NL} < 100$ again in the plane of $\log_{10} g$ and $\log_{10} \lambda$, and for $v/M_p = 10^{-3}$.

The size and position of the excluded region is rather sensitive to the value of the VEV, v. We have varied $\log_{10} v/M_p$ between -1 and -9 to show how the constraints change with v in figure 13. One sees that the width of the excluded region for $\log_{10} g$ grows linearly

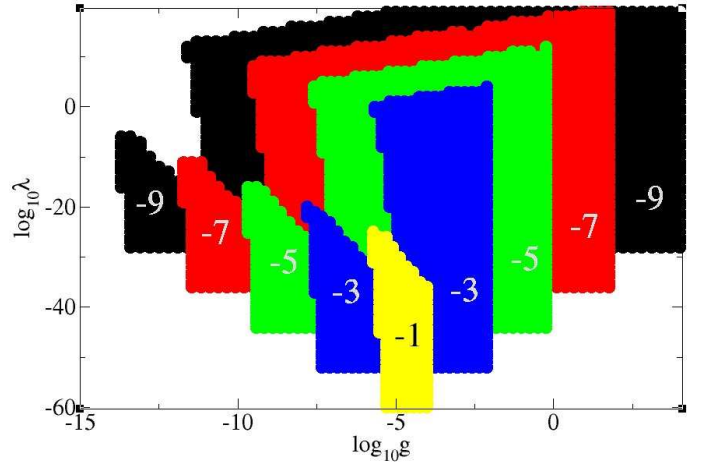with $\log_{10} M_p/v$, so the constraints are strongest at the smallest values of v.

FIG. 13: Regions excluded by spectral distortion for $\log_{10} v/M_p = -1, -3, -5, -7, -9$, as labeled on the figure. Note that for each v, the regions at smaller g are also excluded, but due to nongaussianity (Appendix F).

VII. IMPLICATIONS FOR BRANE-ANTIBRANE INFLATION

We now consider what implications our analysis has for the case of brane-antibrane inflation [55]-[60], which is a string theoretic realization of hybrid inflation. In

this scenario inflation is driven by the interactions between a brane and antibrane which are parallel to the three large dimensions and are initially separated in the extra dimensions of string theory. The brane-antibrane pair experiences an attractive force and move towards each other with inflation ending when the branes collide and annihilate. The role of the inflaton φ is played by the interbrane separation while the field σ is the lightest streched string mode between the brane and antibrane, which develops a tachyonic mass when the separation between brane and antibrane becomes of order the string scale.

The potential of brane-antibrane inflation is not ammenable to the form (9) for several reasons:

- The inflaton potential typically has the Coulombic form $V \sim A/\varphi^n$, rather than the simple form $V \sim m_\varphi^2 \varphi^2$.

- The tachyonic field is complex, rather than real.

- The tachyon potential does not have its minimum at a finite value of the σ field.

- The tachyon field is described by a Dirac-Born-Infeld (DBI) action rather than a simple Klein-Gordon action as we have assumed in this paper.

Here we will assume that these differences do not significantly alter the analysis and leave a more detailed investigation for the future. Specifically, this amounts to assuming that the behavior of the two models will be roughly the same if we match the quadratic parts of their actions, as well as the cross-coupling between the inflaton and the tachyon. Since the timescale for tachyon condensation is given by $\sqrt{-m_\sigma^{-2}}$, one might hope that the results are rather insensitive to the details of how the tachyonic instability saturates at large field values.

In string theory the open string tachyon T between a Dp-brane and antibrane, separated by a distance y is described by the action [61]

$$
\begin{aligned}
S_{\text{tac}} &= -\int d^{p+1}x \sqrt{-g}\,\mathcal{L} \\
\mathcal{L} &= V(T,y)\sqrt{1 + M_s^{-2}g^{\mu\nu}\partial_\mu T \partial_\nu T^\dagger}
\end{aligned}
\tag{76}
$$

where the small $|T|$ expansion of the potential is

$$
V(T,y) = \tau_p \left[1 + \frac{1}{2}\left(\frac{(M_s y)^2}{(2\pi)^2} - \frac{1}{2} \right)|T|^2 \right.
\tag{77}
$$

$$
\left. + \mathcal{O}(|T|^4) + \cdots \right].
\tag{78}
$$

In the above we have neglected the gauge fields and transverse scalars on the branes. The coefficient of the $|T|^4$ term in the potential is order unity and its exact numerical value is unimportant for our estimates since the potential $V(T, y=0)$ is minimized only at infinite T. A potential of the form

$$
V(T, y=0) = \tau_p\, e^{-|T|^2/a^2}
$$

is typical. In the above M_s is the fundamental string mass and τ_p is the Dp-brane tension. In order to obtain order-of-magnitude estimates for the model parameters λ, g, v of (9) it is sufficient to consider only the tachyon part of the action (76) and neglect the inflaton potential. We restrict ourselves to inflation models driven by D3-branes since inflation driven by higher dimensional branes have problems with overclosure of the universe by defect formation [62]. The D3-brane tension is given by

$$
\tau_3 = \frac{M_s^4}{(2\pi)^3 g_s}
\tag{79}
$$

where g_s is the string coupling.

A. The KKLMMT Model

Brane inflation models in torodial compactifications are not realistic because the moduli which describe the size and shape of the extra dimensions were assumed fixed without specifying any mechanism for their stabilization. Recently there has been significant progress [58, 59] in reconciling brane inflation with modulus stabilization using warped geometries with background fluxes for type IIB vacua [63]. For our purposes the most important feature of these compactifications is the presence of strongly-warped throats within the extra dimensions. The compactification geometry within the throat is well approximated by

$$
ds^2 = a(y)^2 g_{\mu\nu}dx^\mu dx^\nu + dy^2 + y^2 d\Omega_5^2
\tag{80}
$$

where y is the distance along the throat, $a(y) = e^{-ky}$ is the throat's warp factor and $d\Omega_5^2$ is the metric on the base space of the corresponding conifold singularity of the underlying Calabi-Yau space [64]. In the subsequent analysis we ignore the base space and treat the geometry as AdS$_5$.

A mobile D3-brane is placed near the UV end of the throat $y = 0$ while an anti-D3-brane remains fixed at some location near the IR end of the throat $y = y_i > 0$. The warp factor $a_i = a(y_i)$ can be easily made much less than unity by a suitable choice of background fluxes and this large warping flattens the inflaton potential. This large warping also suppresses the tachyon potential (effectively reducing the brane tension to $a_i^4 \tau_3$). At quadratic order, the tachyon action takes the form

$$
\begin{aligned}
S = -\int d^4x \sqrt{-g} & \left[a_i^4 \tau_3 + \frac{a_i^4 \tau_3}{2}\left(\frac{M_s^2 y^2}{(2\pi)^2} - \frac{1}{2} \right)|T|^2 \right. \\
& \left. - \frac{a_i^2 \tau_3}{2M_s^2}|\partial T|^2 \right]
\end{aligned}
\tag{81}
$$

The effective values of the couplings can be found by rewriting the action (81) in terms of the canonically normalized fields $\sigma = a_i \sqrt{\tau_3} T / M_s$ and $\varphi = \sqrt{\tau_3} y$ (see equations 3.6, 3.10 or C.1 in [58]), and then matching to the

hybrid inflation potential (9). This gives the correspondence

$$v = \frac{a_i}{\pi^{3/2}} \frac{M_s}{\sqrt{g_s}}$$

$$\lambda = \frac{\pi^3}{2} g_s$$

$$g = a_i \sqrt{2\pi g_s}$$

Before trying to make a general investigation of the parameter space, we will examine the fiducial values which were considered natural in [58]: $a_i = 2.5 \times 10^{-4}$, $g_s = 0.1$, $\tau_3/M_p^4 = 10^{-3}$. The COBE normalization also fixes the inflationary energy scale to be $\Lambda = 1.3 \times 10^{14}$ GeV in this model. One finds that these values imply

$$\frac{v}{M_p} = 10^{-4.2}, \quad \lambda = 10^{0.2}, \quad g = 10^{-3.7} \quad (82)$$

By recomputing the excluded region in the λ-g plane for this value of v/M_p, we find this point to be solidly excluded.

To get out of the excluded region, one should try to achieve smaller values of g or larger values of v/M_p. One can show from the above relations that

$$\frac{v}{M_p} = \left(\frac{8}{\pi^6 g_s}\right)^{1/4} \frac{\Lambda}{M_p} \quad (83)$$

Since Λ is fixed by the COBE normalization, lowering the string coupling is the only way to increase v/M_p. Decreasing g_s also has the desired effect of reducing g. The other parameter we can adjust is the warp factor, but there is not much leeway for decreasing a_i because the density contrast in this model is given by [58]

$$\delta_H = C_1 N_e^{5/6} \left(\frac{\tau_3}{M_p^4}\right)^{1/3} a_i^{4/3} \quad (84)$$

where C_1 is a constant, and $N_e \cong 60$. (Lower values of N_e would require lower values of Λ, which push v down, contrary to what we want.) τ_3 cannot be pushed above the Planck scale, so a_i can only be decreased by at most a factor of $10^{-3/4}$. Although it also goes against the validity of the low-energy effective supergravity theory upon which this model is based, in the interest of being conservative when deriving constraints, we will consider values of the warp factor which are this small. By scanning over g_s and a_i, we find the allowed and excluded regions in the plane of the warped string scale and the string coupling, shown in figure (14). One consequence is an upper bound on the string coupling,

$$g_s < 10^{-4.5} \quad (85)$$

which is stringent, and smaller than normally expected. Our bound differs from those found in [15], which also considered the nongaussian perturbations produced by
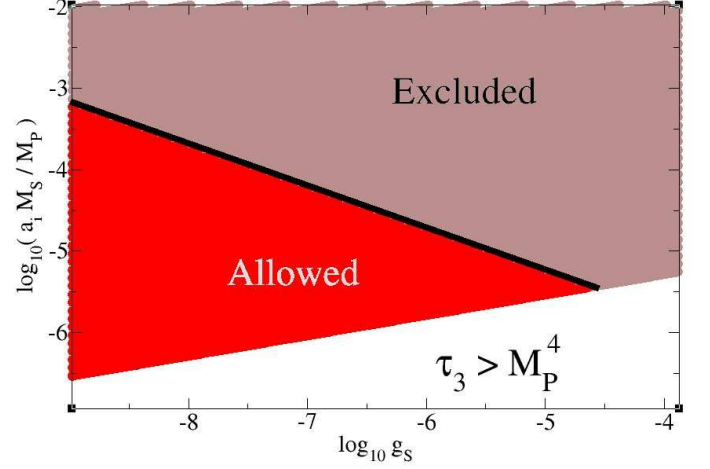

FIG. 14: Allowed and excluded values of the warped string scale and string coupling in the KKLMMT model, due to tachyonic preheating. The fiducial point corresponding to (82) is at $(-1, -3.9)$, off the scale of this graph, but in the excluded region.

tachyonic preheating. Their allowed regions are also expressed in the plane of M_s and g_s, but differ markedly from ours.

A caveat to this result is the potential for complications due to the peculiar form of the tachyon action (76)[8]. The dynamics of the open string tachyon action is different from the usual Klein-Gordon action which our analysis assumes. In the case of (76) the potential is minimized at $T = \infty$ and it takes an infinite amount of time for the condensation to complete (and the quantum mechanical decay of the brane-antibrane system into closed string states invalidates this classical description on a time scale comparable to $(a_i M_s)^{-1}$). Thus the criterion $\langle(\delta\sigma)^2\rangle \sim v^2$ defines the limit of applicability of our analysis, rather than the end of the symmetry breaking process. Moreover, the field theory (76) leads to finite-time divergences in the spatial derivatives $\partial_i T$ near the core of the topological defects where the field stays pinned at $T = 0$ [62, 65], as well as finite-time divergences in the second derivatives $\partial_i \partial_j T$ in regions where $V \to 0$, called caustics [66]. It is not clear how these peculiarities might modify our estimate for f_{NL}, but we plan to investigate this in future work.

---

[8] The inflaton is also described by a DBI action (though the potential does not appear as a prefactor), however, slow roll ensures that it will always be a good approximation to expand the square root to obtain a standard kinetic term. In the case of the tachyon this may not always be a good approximation, particularly during the instability phase.

VIII. CONCLUSIONS

We have carefully calculated the contribution from tachyonic preheating to the spectrum and bispectrum of the curvature perturbation in a model of hybrid inflation, and derived constraints on the model parameters using experimental limits on the power spectrum and nongaussianity in the CMB. The tachyonic contribution to the spectrum, which we parametrized by f_L, has spectral index $n = 4$, and the experimental limit on f_L turns out to always be a stronger constraint than that coming from nongaussianity. Thus we do not expect to see nongaussian signals in future data sets, if hybrid inflation is the underlying model. Rather, one should look for an excess of power at small scales to see the effects we have investigated.

We have studied a simplified model of hybrid inflation, in which the tachyon is a real scalar field. This model is not viable as it stands, because domain walls will be copiously produced at the end of inflation, and they will quickly dominate the universe. We need to assume that there are some small operators in the Lagrangian which violate the symmetry $\sigma \rightarrow -\sigma$, and make the domain walls unstable. Although our results are strictly valid only for hybrid inflation with real fields, we have made a preliminary attempt to apply them to the warped brane-antibrane inflation model, by matching the coefficients of the lowest dimension operators in the corresponding Lagrangians. To the extent that this is valid, we find an interesting limit on the string coupling, $g_s < 10^{-4.5}$.

One achievement of this work is our demonstration that nonlocal contributions to the second order curvature perturbation, as defined herein, vanish when care is taken to consistently include all terms of a given order in the slow-roll expansion. The nonlocal terms which appear at intermediate steps when using the generalized longitudinal gauge have been the bane of some previous attempts to do similar calculations. They should not appear in any physical quantities since they violate causality. Any estimates made using the nonlocal terms are likely to grossly overestimate perturbations on large scales, since these terms are singular at small wave numbers.

Some readers may feel uneasy about an event at the end of inflation being able to affect fluctuations whose wavelength corresponds to the horizon at the beginning of inflation. However, as has been emphasized in [28, 30], there is no violation of causality here. Consider a collection of comoving observers who are within the horizon at the beginning of inflation, but are about to lose causal contact with each other. If they agree to set up perturbations with small wavelengths in their Hubble patches at the end of inflation, this will induce large-scale correlations between different Hubble patches. Since the tachyon field is synchronized across these different Hubble patches it is able to amplify the large-scale perturbations in a coherent way.

One avenue for future study is the more realistic class of models where σ is complex and the defects which form are cosmic strings instead of domain walls. However, this is a much more technically difficult model to study than the case of the real scalar. In the latter case, the VEV of σ remains zero even during the tachyonic preheating phase, because the field rolls in opposite directions on either side of each domain wall, and the VEV averages to zero. In the complex case, the modulus of σ grows uniformly during preheating, and the randomly varying phase is only a gauge degree of freedom. We can no longer decouple the equations for the fluctuations of the metric and the inflationary fields when σ has a VEV. It thus remains an interesting question whether the realistic hybrid inflation model also gets significant nongaussianity from tachyonic preheating [67].

APPENDIX A: Generic Field in deSitter Space

In this appendix we review the solutions of the Klein-Gordon equation in deSitter space [2, 39]. If we consider the limit of deSitter space $\epsilon = \eta = 0$ then φ_0, H are constant and both the tachyon and the inflaton first order fluctuations satisfy an equation of the form

$$\ddot{\xi}_k + 3H\dot{\xi}_k + \left[\frac{k^2}{a^2} + m_\xi^2\right]\xi_k = 0 \qquad \text{(A-1)}$$

or, in terms of conformal time

$$\xi_k'' + 2\mathcal{H}\xi_k' + \left[k^2 + a^2 m_\xi^2\right]\xi_k = 0$$

where $a = e^{Ht} = -1/H\tau$ and $\mathcal{H} = -1/\tau$. Defining $f_k = a\xi_k$ the equation simplifies to

$$f_k'' + \left[k^2 + \frac{1}{\tau^2}\left(\frac{m_\xi^2}{H^2} - 2\right)\right]f_k = 0. \qquad \text{(A-2)}$$

This is the variable in terms of which the action is canonically normalized. Choosing the Bunch-Davies vacuum corresponds to normalizing the solutions so that [39]

$$f_k(\tau) \approx e^{-ik\tau}/\sqrt{2k} \qquad \text{(A-3)}$$

on small scales $k \gg aH$ or equivalently $-k\tau \gg 1$. The general solutions to (A-2) are

$$f_k(\tau) = \sqrt{-\tau}\left[c_1(k)H_\nu^{(1)}(-k\tau) + c_2(k)H_\nu^{(2)}(-k\tau)\right]$$
$$\text{(A-4)}$$

where the order of the Hankel functions is $\nu = \sqrt{9/4 - m_\xi^2/H^2}$. This analysis makes no assumptions

about the size of m_ξ/H and all the formulae we derive are valid for arbitrary complex ν unless otherwise stated. Notice that $-\tau \geq 0$ for all t so that the arguments of the Hankel functions are always positive definite. We recover the desired asymptotics (A-3) on $-k\tau \ll 1$ with the choice

$$c_1(k) = \frac{\sqrt{\pi}}{2} \exp\left[\frac{i\pi}{2}\left(\nu + \frac{1}{2}\right)\right], \quad c_2(k) = 0.$$

The solution becomes

$$f_k(\tau) = \frac{\sqrt{-\pi\tau}}{2} \exp\left[\frac{i\pi}{2}\left(\nu + \frac{1}{2}\right)\right] H_\nu^{(1)}(-k\tau) \quad \text{(A-5)}$$

or, in terms of cosmic time and the original field variable

$$\xi_k(t) = \frac{1}{2}\sqrt{\frac{\pi}{a^3 H}} \exp\left[\frac{i\pi}{2}\left(\nu + \frac{1}{2}\right)\right] H_\nu^{(1)}\left(\frac{k}{aH}\right). \quad \text{(A-6)}$$

To get an intuitive feel for this initial condition we consider the asymptotic behaviour of (A-5) on large scales:

$$f_k(\tau) \rightarrow \frac{\sqrt{-\pi\tau}}{2} \exp\left[\frac{i\pi}{2}\left(\nu + \frac{1}{2}\right)\right] \quad \text{(A-7)}$$
$$\times \left[\frac{1}{\Gamma(\nu+1)}\left(-\frac{k\tau}{2}\right)^\nu - i\frac{\Gamma(\nu)}{\pi}\left(-\frac{k\tau}{2}\right)^{-\nu}\right]$$

as $-k\tau \to 0$. To simplify further we should focus on either $m_\xi/H > 3/2$ or $m_\xi/H < 3/2$.

We consider a field which is heavy during inflation $m_\xi \gg H$ so that $\nu \approx im_\xi/H$ is pure imaginary. How do the two terms in the square braces of (A-7) compare? The functions $(-k\tau)^{im_\xi/H}$ are oscillatory so the relative magnitude of these two terms depends only on the gamma function prefactors. Using the results (for β real and arbitrary complex z)

$$\Gamma(1+z) = z\Gamma(z), \quad |\Gamma(1+i\beta)| = \sqrt{\frac{\pi\beta}{\sinh(\pi\beta)}}$$

from the theory of gamma functions we find that for $m_\xi/H \gg 1$

$$\left|\frac{1}{\Gamma(1+im_\xi/H)}\right| \sim \sqrt{\frac{H}{2\pi m_\xi}} \exp\left(\frac{\pi m_\xi}{2H}\right),$$

$$|\Gamma(im_\xi/H)| \sim \sqrt{\frac{2\pi H}{m_\xi}} \exp\left(-\frac{\pi m_\xi}{2H}\right)$$

so that the first term in the square braces on the second line of (A-7) dominates at large m_ξ/H. Going back to the original variable we find

$$|\xi_k| \sim a^{-3/2} \frac{1}{2^{3/2}\sqrt{m_\xi}} \quad \text{for } m_\xi \gg H$$

on large scales.

We consider now a field which is light during inflation $m_\xi \ll H$. In this case $\nu \approx 3/2$ is pure real and the first term in the square braces in (A-7) goes to zero. We recover the scale invariant spectrum

$$|\xi_k| \sim \frac{H}{\sqrt{2k^3}} \quad \text{for } m_\xi \ll H \quad \text{(A-8)}$$

on large scales, as expected.

Notice that for any value of m_ξ/H we have the small scale asymptotics

$$|\xi_k| \sim a^{-1}\frac{1}{\sqrt{2k}}$$

by constuction.

During inflation $a(t)$ grows exponentially so that the large scale fluctuations of any massive field are damped as $a^{-3/2}$ while the large scale fluctuations of any light field are approximately constant. On small scales the fluctuations of all fields get damped as a^{-1}.

APPENDIX B: Perturbed Einstein Equations and the Master Equation

Using Maple we have carefully verified the results of [32] for the perturbed Einstein equations and the master equation. Here we briefly review those results relevent for the computation of $\zeta^{(2)}$. We present only the $\delta^{(2)}G_0^0 = \kappa^2\delta^{(2)}T_0^0$, $\partial_i\delta^{(2)}G_0^i = \kappa^2\partial_i\delta^{(2)}T_0^i$ and $\delta_j^i\delta^{(2)}G_i^j = \kappa^2\delta_j^i\delta^{(2)}T_i^j$ equations since the second order vector and tensor fluctuations decouple from this system. In the case that $\sigma_0 = 0$ the second order tachyon fluctation $\delta^{(2)}\sigma$ decouples from the inflaton and gravitational fluctuations. Analogously to the first order fluctuations, the Klein-Gordon equation for the inflaton fluctuation at second order $\delta^{(2)}\varphi$ is not necessary to close the system of equations. In the subsequent text we sometimes insert the slow roll parameters ϵ and η explicitly though we make no assumption that they are small. We also introduce the shorthand notation $m_\varphi^2 = \partial^2 V/\partial\varphi^2$ and $m_\sigma^2 = \partial^2 V/\partial\sigma^2$ and assume that $\partial^2 V/\partial\sigma\partial\varphi = 0$.

The second order $(0,0)$ equation is

$$3\mathcal{H}\psi'^{(2)} + (3-\epsilon)\mathcal{H}^2\phi^{(2)} - \partial^k\partial_k\psi^{(2)}$$
$$= -\frac{\kappa^2}{2}\left[\varphi_0'\delta^{(2)}\varphi' + a^2\frac{\partial V}{\partial\varphi}\delta^{(2)}\varphi\right] + \Upsilon_1 \quad \text{(B-1)}$$

where Υ_1 is constructed entirely from first order quantities. Dividing Υ_1 into inflaton and tachyon contributions we have

$$\Upsilon_1 = \Upsilon_1^\varphi + \Upsilon_1^\sigma$$

where

$$\Upsilon_1^\varphi = 4(3-\epsilon)\mathcal{H}^2\left(\phi^{(1)}\right)^2 + 2\kappa^2\varphi_0'\phi^{(1)}\delta^{(1)}\varphi'$$
$$- \frac{\kappa^2}{2}\left(\delta^{(1)}\varphi'\right)^2 - \frac{\kappa^2}{2}a^2 m_\varphi^2\left(\delta^{(1)}\varphi\right)^2 - \frac{\kappa^2}{2}\left(\vec{\nabla}\delta^{(1)}\varphi\right)^2$$
$$+ 8\phi^{(1)}\partial^k\partial_k\phi^{(1)} + 3\left(\phi'^{(1)}\right)^2 + 3\left(\vec{\nabla}\phi^{(1)}\right)^2 \quad \text{(B-2)}$$

and

$$\Upsilon_1^\sigma = -\frac{\kappa^2}{2}\left[\left(\delta^{(1)}\sigma'\right)^2 + \left(\vec{\nabla}\delta^{(1)}\sigma\right)^2\right.$$
$$\left. + \; a^2 m_\sigma^2\left(\delta^{(1)}\sigma\right)^2\right]. \tag{B-3}$$

The divergence of the second order $(0,i)$ equation is

$$\partial^k\partial_k\left[\psi'^{(2)} + \mathcal{H}\phi^{(2)}\right] = \frac{\kappa^2}{2}\varphi_0'\partial^k\partial_k\delta^{(2)}\varphi + \Upsilon_2 \tag{B-4}$$

where $\Upsilon_2 = \Upsilon_2^\varphi + \Upsilon_2^\sigma$ is constructed entirely from first order quantities. The inflaton part is

$$\Upsilon_2^\varphi = 2\kappa^2\varphi_0'\partial_i\left(\phi^{(1)}\partial^i\delta^{(1)}\varphi\right) + \kappa^2\partial_i\left(\delta^{(1)}\varphi'\partial^i\delta^{(1)}\varphi\right)$$
$$- \; 8\partial_i\left(\phi^{(1)}\partial^i\phi'^{(1)}\right) - 2\partial_i\left(\phi'^{(1)}\partial^i\phi^{(1)}\right) \tag{B-5}$$

and the tachyon part is

$$\Upsilon_2^\sigma = \kappa^2\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right). \tag{B-6}$$

The trace of the second order (i,j) equation is

$$3\psi''^{(2)} + \partial^k\partial_k\left[\phi^{(2)} - \psi^{(2)}\right] + 6\mathcal{H}\psi'^{(2)}$$
$$+ \; 3\mathcal{H}\phi'^{(2)} + 3(3-\epsilon)\mathcal{H}^2\phi^{(2)}$$
$$= \; \frac{3\kappa^2}{2}\left[\varphi_0'\delta^{(2)}\varphi' - a^2\frac{\partial V}{\partial\varphi}\delta^{(2)}\varphi\right] + \Upsilon_3 \tag{B-7}$$

where $\Upsilon_3 = \Upsilon_3^\varphi + \Upsilon_3^\sigma$ is constructed entirely from first order quantities. The inflaton part is

$$\Upsilon_3^\varphi = 12(3-\epsilon)\mathcal{H}^2\left(\phi^{(1)}\right)^2 - 6\kappa^2\varphi_0'\phi^{(1)}\delta^{(1)}\varphi'$$
$$+ \; \frac{3\kappa^2}{2}\left(\delta^{(1)}\varphi'\right)^2 - \frac{3\kappa^2}{2}a^2 m_\varphi^2\left(\delta^{(1)}\varphi\right)^2 - \frac{\kappa^2}{2}\left(\vec{\nabla}\delta^{(1)}\varphi\right)^2$$
$$+ \; 3\left(\phi'^{(1)}\right)^2 + 8\phi^{(1)}\partial^k\partial_k\phi^{(1)} + 24\mathcal{H}\phi^{(1)}\phi'^{(1)}$$
$$+ \; 7\left(\vec{\nabla}\phi^{(1)}\right)^2 \tag{B-8}$$

and the tachyon part is

$$\Upsilon_3^\sigma = \kappa^2\left[\frac{3}{2}\left(\delta^{(1)}\sigma'\right)^2 - \frac{1}{2}\left(\vec{\nabla}\delta^{(1)}\sigma\right)^2\right.$$
$$\left. - \; \frac{3}{2}a^2 m_\sigma^2\left(\delta^{(1)}\sigma\right)^2\right] \tag{B-9}$$

We now proceed to derive the master equation. Adding (B-1) to the inverse laplacian of the time derivative of (B-4) and then using (B-4) to eliminate $\delta^{(2)}\varphi$ yields

$$\psi''^{(2)} - (1 + 2\epsilon - 2\eta)\mathcal{H}\psi'^{(2)} + \mathcal{H}\phi'^{(2)}$$
$$- \; 2(2\epsilon - \eta)\mathcal{H}^2\phi^{(2)} - \partial^k\partial_k\psi^{(2)} = \Upsilon_1 + \triangle^{-1}\Upsilon_2'$$
$$- \; 2(2 + \epsilon - \eta)\mathcal{H}\triangle^{-1}\Upsilon_2. \tag{B-10}$$

Notice that we have decoupled $\psi^{(2)}$ and $\phi^{(2)}$ from the inflaton perturbation $\delta^{(2)}\varphi$. It remains now to express $\psi^{(2)}$ in terms of $\phi^{(2)}$. To this end we subtract the inverse laplacian of (B-4) from (B-7) and again use (B-4) to eliminate $\delta^{(2)}\varphi$ which gives

$$\partial^k\partial_k\left[\phi^{(2)} - \psi^{(2)}\right] = \Upsilon_3 - 3\triangle^{-1}\Upsilon_2' - 6\mathcal{H}\triangle^{-1}\Upsilon_2$$

or, equivalently

$$\psi^{(2)} = \phi^{(2)} - \triangle^{-1}\gamma. \tag{B-11}$$

Following the notation of [32] we have defined

$$\gamma = \Upsilon_3 - 3\triangle^{-1}\Upsilon_2' - 6\mathcal{H}\triangle^{-1}\Upsilon_2 \tag{B-12}$$

which can be split into inflaton and tachyon components $\gamma = \gamma_\varphi + \gamma_\sigma$ in an obvious fashion. Our Υ_2 is related to the quantities α, β defined in [32] by

$$\Upsilon_2 = \kappa^2\beta - \alpha.$$

Now, using (B-11) to eliminate $\psi^{(2)}$ from (B-10) gives the master equation

$$\phi''^{(2)} + 2(\eta - \epsilon)\mathcal{H}\phi'^{(2)} + \left[2(\eta - 2\epsilon)\mathcal{H}^2 - \partial^k\partial_k\right]\phi^{(2)}$$
$$= \; \Upsilon_1 + \triangle^{-1}\Upsilon_2' - 2(2 + \epsilon - \eta)\mathcal{H}\triangle^{-1}\Upsilon_2 - \gamma$$
$$- \; (1 + 2\epsilon - 2\eta)\mathcal{H}\triangle^{-1}\gamma' + \triangle^{-1}\gamma''$$

Inserting explicitly the expression for γ (B-12) this can be written as

$$\phi''^{(2)} + 2(\eta - \epsilon)\mathcal{H}\phi'^{(2)} + \left[2(\eta - 2\epsilon)\mathcal{H}^2 - \partial^k\partial_k\right]\phi^{(2)} = J$$

where the source is

$$J = \Upsilon_1 - \Upsilon_3 + 4\triangle^{-1}\Upsilon_2' + 2(1 - \epsilon + \eta)\mathcal{H}\triangle^{-1}\Upsilon_2$$
$$+ \; \triangle^{-1}\gamma'' - (1 + 2\epsilon - 2\eta)\mathcal{H}\triangle^{-1}\gamma'. \tag{B-13}$$

We can split the source into tachyon and inflaton contributions $J = J^\varphi + J^\sigma$ in the obvious manner, by taking the tachyon and inflaton parts of $\Upsilon_1, \Upsilon_2, \Upsilon_3, \gamma$.

We now derive some results concering the tachyon source terms which will be useful in the text. First we consider the tachyon contribution to γ (B-12):

$$\gamma_\sigma = \Upsilon_3^\sigma - 3\triangle^{-1}\partial_\tau\Upsilon_2^\sigma - 6\mathcal{H}\triangle^{-1}\Upsilon_2^\sigma.$$

Using equations (B-6) and (B-9) we can write this as

$$\gamma_\sigma = \kappa^2\left[\frac{3}{2}(\delta^{(1)}\sigma')^2 - \frac{1}{2}\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma - \frac{3}{2}a^2 m_\sigma^2(\delta^{(1)}\sigma)^2\right]$$
$$- \; 3\kappa^2\triangle^{-1}\partial_\tau\partial_i(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma)$$
$$- \; 6\kappa^2\mathcal{H}\triangle^{-1}\partial_i(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma) \tag{B-14}$$

We now write γ_σ as

$$\gamma_\sigma = \kappa^2\triangle^{-1}\left[\frac{3}{2}\partial^k\partial_k(\delta^{(1)}\sigma')^2 - \frac{1}{2}\partial^k\partial_k(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma)\right.$$
$$- \; \frac{3}{2}a^2 m_\sigma^2\partial^k\partial_k(\delta^{(1)}\sigma)^2 - 3\partial_\tau\partial_i(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma)$$
$$\left. - \; 6\mathcal{H}\partial_i(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma)\right]$$

and, after some algebra, we have

$$
\gamma_\sigma = \kappa^2 \triangle^{-1} \left[-\frac{1}{2} \partial^k \partial_k (\partial_i \delta^{(1)} \sigma \partial^i \delta^{(1)} \sigma) \right.
$$
$$
- 3\partial_i \left(\delta^{(1)} \sigma'' + 2\mathcal{H} \delta^{(1)} \sigma' + a^2 m_\sigma^2 \delta^{(1)} \sigma \right) \partial^i \delta^{(1)} \sigma
$$
$$
\left. - 3 \left(\delta^{(1)} \sigma'' + 2\mathcal{H} \delta^{(1)} \sigma' + a^2 m_\sigma^2 \delta^{(1)} \sigma \right) \partial^k \partial_k \delta^{(1)} \sigma \right]
$$

The last two lines can be simplified using the equation of motion for the tachyon fluctuation

$$
\delta^{(1)} \sigma'' + 2\mathcal{H} \delta^{(1)} \sigma + a^2 m_\sigma^2 \delta^{(1)} \sigma = \partial^k \partial_k \delta^{(1)} \sigma
$$

which gives

$$
\gamma_\sigma = \kappa^2 \triangle^{-1} \left[-\frac{1}{2} \partial^k \partial_k \left(\partial_i \delta^{(1)} \sigma \partial^i \delta^{(1)} \sigma \right) \right.
$$
$$
\left. - 3\partial_i (\partial^k \partial_k \delta^{(1)} \sigma) \partial^i \delta^{(1)} \sigma - 3\partial_i \partial^i \delta^{(1)} \sigma \partial^k \partial_k \delta^{(1)} \sigma \right]
$$
$$
= \kappa^2 \triangle^{-1} \left[-\frac{1}{2} \partial^k \partial_k \left(\partial_i \delta^{(1)} \sigma \partial^i \delta^{(1)} \sigma \right) \right.
$$
$$
\left. - 3\partial_i \left(\partial^k \partial_k \delta^{(1)} \sigma \partial^i \delta^{(1)} \sigma \right) \right] \qquad \text{(B-15)}
$$

This result has also been derived in [32]. We now consider the tachyon contribution to the source. We take

$$
J^\sigma = \Upsilon_1^\sigma - \Upsilon_3^\sigma + 4\triangle^{-1} \partial_\tau \Upsilon_2^\sigma + 2(1 - \epsilon + \eta)\mathcal{H}\triangle^{-1}\Upsilon_2^\sigma
$$
$$
+ \triangle^{-1} \gamma_\sigma'' - (1 + 2\epsilon - 2\eta)\mathcal{H}\triangle^{-1}\gamma_\sigma'
$$

and, using (B-3), (B-6) and (B-9), we have

$$
J^\sigma(\tau, \vec{x}) = a^2 \kappa^2 m_\sigma^2 \left(\delta^{(1)} \sigma \right)^2 - 2\kappa^2 \left(\delta^{(1)} \sigma' \right)^2
$$
$$
+ 2\kappa^2 \mathcal{H}(1 + \eta - \epsilon)\triangle^{-1} \partial_i \left(\delta^{(1)} \sigma' \partial^i \delta^{(1)} \sigma \right)
$$
$$
+ 4\kappa^2 \triangle^{-1} \partial_\tau \partial_i \left(\delta^{(1)} \sigma' \partial^i \delta^{(1)} \sigma \right)
$$
$$
- \mathcal{H}(1 + 2\epsilon - 2\eta)\triangle^{-1} \gamma_\sigma' + \triangle^{-1} \gamma_\sigma''. \quad \text{(B-16)}
$$

APPENDIX C: The Inflaton Contribution to $\zeta^{(2)}$

In this appendix we consider the calculation of the inflaton part of the second order curvature perturbation $\zeta_\varphi^{(2)}$ using results from appendix B and following closely the calculation of the tachyon part of the second order curvature perturbation which is discussed in subsection IV D.

The first step to computing $\zeta_\varphi^{(2)}$ is to use the Green function (44) to eliminate the dependence of $\zeta_\varphi^{(2)}$ on $\phi^{(2)}$ (from the first line of (48) which is equivalent to (53)) in favour of $\phi^{(1)}$, $\delta^{(1)}\varphi$. This procedure follows closely the analysis of subsection IV D and in particular (56) still holds though here we are interested only in the inflaton part of the souce J^φ.

Having removed the explicit dependence of $\zeta^{(2)}$ on the second order metric perturbation $\phi^{(2)}$ we next eliminate

$\phi^{(1)}, \delta^{(1)}\varphi$ in favour of $\zeta^{(1)}$. Using the solution (22) and the first order constraint equation (15) one may verify that on large scales

$$
\zeta^{(1)} \cong -\phi^{(1)} - \frac{\varphi_0'}{\mathcal{H}} \delta^{(1)}\varphi \cong -\frac{1}{\epsilon}\phi^{(1)}
$$

and

$$
\delta^{(1)}\varphi \cong \frac{\varphi_0'}{\mathcal{H}} \frac{1}{\epsilon}\phi^{(1)} \cong -\frac{\varphi_0'}{\mathcal{H}}\zeta^{(1)}.
$$

On large scales the first order curvature perturbation is approximately constant since [42]

$$
\zeta'^{(1)} = \frac{\partial^k \partial_k}{\epsilon \mathcal{H}} \phi^{(1)}
$$

using the fact that $\sigma_0 = 0$ (so that there are no anisotropic stresses, whose absence guarantees the conservation of $\zeta^{(1)}$ on super-Hubble scales). Thus on large scales we have

$$
\delta^{(1)}\varphi' \cong -(2\epsilon - \eta)\varphi_0'\zeta^{(1)}.
$$

It is straightforward to compute the last three lines of (48) using the fact that $Q_\varphi^{(1)} \approx -\varphi_0'\zeta^{(1)}/\mathcal{H}$ and $Q_\varphi'^{(1)} \approx -(2\epsilon - \eta)\varphi_0'\zeta^{(1)}$ on large scales. The result is

$$
\zeta^{(2)} \ni \left[\frac{1}{3} \left(\frac{a\, m_\varphi}{\mathcal{H}} \right)^2 + 2 + 2\epsilon - 2\eta \right] (\zeta^{(1)})^2 \qquad \text{(C-1)}
$$

where we have dropped terms which are higher order in slow roll parameters or which contain gradients. Notice that the quantity $(a\, m_\varphi/\mathcal{H})^2$ is first order in the slow roll expansion because

$$
\eta \approx \frac{1}{\kappa^2} \frac{1}{V} \frac{\partial^2 V}{\partial \varphi^2} = \frac{1}{\kappa^2} \frac{m_\varphi^2}{V} \approx \frac{1}{3} \left(\frac{a\, m_\varphi}{\mathcal{H}} \right)^2.
$$

To (C-1) we must add the contribution coming from the fist line of (48) which can be written explicitly in terms of first order quantities using equations (53) and (56). Here we consider only the particular solution for $\phi^{(2)}$ due to the inflaton source J^φ.

In order to compute J^φ we first study the quantities $\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \gamma$ which are defined in appendix B. On large scales and to leading order in slow roll we have

$$
\Upsilon_1^\varphi \approx \left[12\epsilon^2 - \epsilon \left(\frac{a\, m_\varphi}{\mathcal{H}} \right)^2 \right] \mathcal{H}^2 (\zeta^{(1)})^2.
$$

The quantity $\triangle^{-1}\Upsilon_2^\varphi$ can be written as (see equation 39 of [32])

$$
\triangle^{-1}\Upsilon_2^\varphi = \frac{\kappa^2}{2}(\epsilon - \eta)\mathcal{H}(\delta^{(1)}\varphi)^2 + 3\mathcal{H}(\phi^{(1)})^2 - 2\phi^{(1)}\phi'^{(1)}
$$
$$
+ \frac{2}{\varphi_0'} \triangle^{-1} \left(\partial_i \partial^k \partial_k \phi^{(1)} \partial^i \delta^{(1)}\varphi \right.
$$
$$
\left. + \partial^k \partial_k \phi^{(1)} \partial_i \partial^i \delta^{(1)}\varphi \right).
$$

The inverse laplacians on the last two lines contribute only to the momentum dependence of f_{NL}^φ which we neglect. On large scales and in the slow roll limit we have

$$\triangle^{-1}\Upsilon_2^\varphi \approx (4\epsilon^2 - \epsilon\eta)\mathcal{H}(\zeta^{(1)})^2 + \cdots$$

where the $\cdots$ denotes momentum dependent terms. On large scales and in the slow roll limit we also have

$$\Upsilon_3^\varphi \approx \left[36\epsilon^2 - 3\epsilon\left(\frac{a\, m_\varphi}{\mathcal{H}}\right)^2\right]\mathcal{H}^2(\zeta^{(1)})^2.$$

The quantity Υ_4 defined in (54) depends only on the inflaton fluctuation and is given by

$$\Upsilon_4 = \Upsilon_4^\varphi \approx (6\epsilon^2 - 2\epsilon\eta)\frac{\varphi_0'}{\mathcal{H}}(\zeta^{(1)})^2$$

on large scales and in the slow roll limit. Using these results one may readily verify that $\gamma_\varphi/\mathcal{H}^2$ is third order in slow roll parameters

$$\gamma_\varphi \approx \mathcal{O}(\epsilon^3)\mathcal{H}^2(\zeta^{(1)})^2.$$

Using these results one may verify that the only term in (53) which contributes at lowest order in slow roll parameters is the term proportional to $\Upsilon_1^\varphi/\epsilon\mathcal{H}^2$. Thus, the contribution to the second order curvature perturbation due to the first line of (48) is

$$\zeta_\varphi^{(2)} \ni \left[4\epsilon - \frac{1}{3}\left(\frac{a\, m_\varphi}{\mathcal{H}}\right)^2\right](\zeta^{(1)})^2.$$

Adding all the contributions together we find

$$\zeta_\varphi^{(2)} \approx (2 - 2\eta + 6\epsilon)(\zeta^{(1)})^2.$$

The contribution $2(\zeta^{(1)})^2$ stems from using the Malik and Wands [41] definition of the second order curvature perturbation. It can be related to the definition of Lyth and Rodriguez [46] (which also agrees with Maldacena [7]) using

$$\zeta^{(2)} = \zeta_{LR}^{(2)} + 2(\zeta^{(1)})^2.$$

The Lyth-Rodriguez curvature perturbation, due to the inflaton up to second order, can thus be written as

$$\zeta_{LR}^\varphi = \zeta^{(1)} - \frac{3}{5}f_{NL}^\varphi(\zeta^{(1)})^2$$

where

$$f_{NL}^\varphi = \frac{5}{6}(2\eta - 6\epsilon).$$

In writing ζ_{LR}^φ above we have suppressed the homogeneous $k = 0$ mode of ζ which should be subtracted to ensure that $\langle\zeta\rangle = 0$. This result differs from previous studies [7, 47] by a factor of two. The calculation of [7, 47] takes into account the effect of nonlinear evolution aswell as the effect of computing the bispectrum in the vacuum of the interacting theory, as opposed to the vacuum of the free theory. Our calculation does not consider the effect of the change in vacuum which is the same order of magnitude. Thus we should not expect to reproduce exactly the results of [7, 47]. The change in vacuum will not change the calculation of the tachyon part of the curvature perturbation $\zeta_\sigma^{(2)}$ since $\zeta^{(1)}$ does not depend on $\delta^{(1)}\sigma$ and hence contributions to ζ_σ due to the change in vacuum will be higher than second order in perturbation theory.

APPENDIX D: Fourier Transforms, Mode Functions and Inverse Laplacians

We define the Fourier transform of some first order quantity $\delta f(t, \vec{x})$ by

$$\delta f(t, \vec{x}) = \int\frac{d^3k}{(2\pi)^{3/2}}e^{i\vec{k}\cdot\vec{x}}\delta\tilde{f}_k(t) \qquad \text{(D-1)}$$

$$\delta\tilde{f}_{\vec{k}}(t) = \int\frac{d^3x}{(2\pi)^{3/2}}e^{-i\vec{k}\cdot\vec{x}}\delta f(t, \vec{x})$$

where the Hermiticity of $\delta f(t, \vec{x})$ implies that $\delta\tilde{f}_k(t) = \delta\tilde{f}_{-k}(t)^\dagger$ so that we can define

$$\delta\tilde{f}_k(t) = a_k\xi_k(t) + a_{-k}^\dagger\xi_{-k}(t)^\star \qquad \text{(D-2)}$$

where a_k is an operator and $\delta f_k(t)$ is a c-number valued mode function. We then re-write the Fourier transform as

$$\delta f(t, x) = \int\frac{d^3k}{(2\pi)^{3/2}}\left[a_k\xi_k(t)e^{ikx} + a_{-k}^\dagger\xi_{-k}(t)^\star e^{ikx}\right]$$

$$= \int\frac{d^3k}{(2\pi)^{3/2}}\left[a_k\xi_k(t)e^{ikx} + a_k^\dagger\xi_k(t)^\star e^{-ikx}\right].$$

In this form it is clear that a_k is the usual creation operator satisfying

$$\left[a_k, a_{k'}^\dagger\right] = \delta^{(3)}(\vec{k} - \vec{k'})$$

and one should expect

$$\xi_k(\tau) \approx \frac{1}{\sqrt{2k}}e^{-ik\tau}$$

on small scales, which corresponds to the Bunch-Davies vacuum $a_k|0\rangle = 0$. This is consistent with the usual definition of the power spectrum in terms of the two-point function

$$\langle 0|\delta f^2(t, \vec{x})|0\rangle = \int\frac{d^3k}{(2\pi)^3}|\xi_k(t)|^2$$

$$= \int\frac{dk}{k}\mathcal{P}_f(k)$$

so that

$$\mathcal{P}_f(k) = \frac{k^3}{2\pi^2}|\xi_k(t)|^2.$$

We now discuss the Fourier transform of the tachyon source. Typical terms in the source have the form

$$
\begin{aligned}
J^{(1)}(t,\vec{x}) &= b(t)\delta f(t,\vec{x})\delta g(t,\vec{x}), \\
J^{(2)}(t,\vec{x}) &= b(t)\triangle^{-1}\left[\delta f(t,\vec{x})\delta g(t,\vec{x})\right], \\
J^{(3)}(t,\vec{x}) &= b(t)\triangle^{-2}\left[\delta f(t,\vec{x})\delta g(t,\vec{x})\right]
\end{aligned}
$$

where δf, δg are some first order quantities and b is constructed from zeroth order quantities. The Green's function for the laplacian is defined as appropriate in the absence of boundary surfaces

$$
(\triangle^{-1}f)(t,\vec{x}) = \int d^3x'\, G(\vec{x}-\vec{x}')f(t,\vec{x}')
$$

where

$$
G(\vec{x}-\vec{x}') = -\frac{1}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|}.
$$

In Fourier space we have

$$
\begin{aligned}
G(x,x') &= \int \frac{d^3k}{(2\pi)^{3/2}}e^{ikx}G_k(x') \\
G_k(x') &= -\frac{1}{(2\pi)^{3/2}}\frac{e^{-ikx'}}{k^2}.
\end{aligned}
$$

In Fourier space the souce terms contain convolutions

$$
\begin{aligned}
J_k^{(1)}(t) &= b(t)(\delta\tilde{f}\star\delta\tilde{g})_k(t), \\
J_k^{(2)}(t) &= -b(t)\frac{1}{k^2}(\delta\tilde{f}\star\delta\tilde{g})_k(t), \\
J_k^{(3)}(t) &= b(t)\frac{1}{k^4}(\delta\tilde{f}\star\delta\tilde{g})_k(t).
\end{aligned}
$$

where $\tilde{f}_k$, $\tilde{g}_k$ are operator valued Fourier transforms defined as in (D-1). These may be related to the mode functions as (D-2). Finally, we have defined convolution by

$$
(\delta\tilde{f}\star\delta\tilde{g})_{\vec{k}}(t) = \int\frac{d^3k'}{(2\pi)^{3/2}}\delta\tilde{f}_{\vec{k}'}(t)\delta\tilde{g}_{\vec{k}-\vec{k}'}(t)
$$

APPENDIX E: Construction of The Tachyon Curvature Perturbation

In this appendix we include technical details about the contruction of $\zeta_\sigma^{(2)}$ using the Green function for the master equation. For clarity we repeat some details which are included in the text. We consider only contributions to the equations which depend on the tachyon field and we remind the reader that $\zeta^{(1)}$ is independent of σ. The second order curvature perturbation is (see (48-50))

$$
\begin{aligned}
\zeta^{(2)} &= \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\varphi_0' Q_\varphi'^{(2)} + a^2\frac{\partial V}{\partial\varphi}Q_\varphi^{(2)}\right] \\
&+ \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\left(\delta^{(1)}\sigma'\right)^2 + a^2 m_\sigma^2\left(\delta^{(1)}\sigma\right)^2\right] \\
&+ \text{ inflaton contributions} \qquad\qquad\qquad \text{(E-1)}
\end{aligned}
$$

where the second order Sasaki-Mukhanov variable is

$$
\begin{aligned}
Q_\varphi^{(2)} &= \delta^{(2)}\varphi + \frac{\varphi_0'}{\mathcal{H}}\psi^{(2)} \\
&+ \text{ inflaton contributions} \qquad\qquad \text{(E-2)}
\end{aligned}
$$

Inserting (E-2) into (E-1) and using the $(0,0)$ Einstein equation (B-1) to eliminate the contribution $\varphi_0'\delta^{(2)}\varphi' + a^2\partial V/\partial\varphi\delta^{(2)}\varphi$ gives

$$
\begin{aligned}
\zeta_\sigma^{(2)} &= -\frac{\phi'^{(2)}}{\epsilon\mathcal{H}} - \left(\frac{1}{\epsilon}+1\right)\phi^{(2)} + \frac{1}{3-\epsilon}\frac{\partial^k\partial_k\phi^{(2)}}{\epsilon\mathcal{H}^2} \\
&+ \frac{1}{\epsilon\mathcal{H}}\triangle^{-1}\gamma_\sigma' + \triangle^{-1}\gamma_\sigma - \frac{1}{3-\epsilon}\frac{1}{\epsilon\mathcal{H}^2}\gamma_\sigma \quad \text{(E-3)}
\end{aligned}
$$

where we have also eliminated $\psi^{(2)}$ in favour of $\phi^{(2)}$ using (B-11). Notice that using (B-1) introduces a term proportional to Υ_1^σ to the curvature perturbation which cancels the contribution proportional to $(\delta^{(1)}\sigma')^2 + a^2 m_\sigma^2(\delta^{(1)}\sigma)^2$ on the second line of (E-1) up to a gradient term $(\vec{\nabla}\delta^{(1)}\sigma)^2$ which can be neglected on large scales. The result (E-3) is valid only on large scales but we have not yet assumed slow roll. In (E-3) it is understood that $\phi^{(2)}$ denotes only the particular solution due to the tachyon source, J^σ. We now use the Green function (44) to solve for $\phi^{(2)}$ in terms of the tachyon source (B-16). We work only to leading order in slow roll parameters. We also work in the large scale limit. To lowest order in ϵ, η and up to order k^2 we can write

$$
\begin{aligned}
-\frac{\phi_k'^{(2)}}{\epsilon\mathcal{H}} &- \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} = \\
&\frac{1}{\epsilon}\int_{\tau_i}^0 d\tau'\Theta(\tau-\tau')J_k^\sigma(\tau') \\
&\times\left[\tau' + \left(-\frac{1}{6}\tau'^3 + \frac{5}{6}\tau^2\tau' - \frac{2}{3}\tau^3\right)k^2\right] \quad \text{(E-4)}
\end{aligned}
$$

using the Green function (44) and the relation (45). Equation (E-4) allows us to eliminate the dependence of $\zeta_\sigma^{(2)}$ on $\phi^{(2)}$. At leading order in slow roll parameters the tachyon source is (see (B-13))

$$
\begin{aligned}
J^\sigma &= \Upsilon_1^\sigma - \Upsilon_3^\sigma + 4\triangle^{-1}\partial_\tau\Upsilon_2^\sigma + 2\mathcal{H}\triangle^{-1}\Upsilon_2^\sigma \\
&+ \triangle^{-1}\partial_\tau^2\gamma_\sigma - \mathcal{H}\triangle^{-1}\partial_\tau\gamma_\sigma. \qquad\qquad \text{(E-5)}
\end{aligned}
$$

We must now insert (E-5) into (E-4), perform numerous integrations by parts, and then insert this result into (E-3). We evaluate term-by-term the last line of (E-4). The first contribution (the term proportional to k^0) is

$$
-\frac{\phi_k'^{(2)}}{\epsilon\mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2\phi_k^{(2)}}{3\epsilon\mathcal{H}^2} \ni -\frac{1}{\epsilon}\int_{\tau_i}^\tau d\tau'\frac{J^\sigma(\tau')}{\mathcal{H}(\tau')}.
$$

Inserting (E-5), noting that $\mathcal{H}(\tau') = -1/\tau'$ at leading

order in slow roll and integrating by parts gives

$$
\begin{aligned}
- \frac{\phi_k'^{(2)}}{\epsilon \mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2 \phi_k^{(2)}}{3\epsilon \mathcal{H}^2} &\ni -\frac{1}{\epsilon} \int_{\tau_i}^{\tau} d\tau' \frac{J^\sigma(\tau')}{\mathcal{H}(\tau')} \\
&\cong \frac{1}{\epsilon} \int_{\tau_i}^{\tau} d\tau' \left[-\frac{\Upsilon_1^\sigma}{\mathcal{H}(\tau')} + \frac{\Upsilon_3^\sigma}{\mathcal{H}(\tau')} - 6\triangle^{-1}\Upsilon_2^\sigma \right] \\
&+ \frac{1}{\epsilon} \left[-4\frac{\triangle^{-1}\Upsilon_2^\sigma}{\mathcal{H}} - \frac{\triangle^{-1}\gamma_\sigma'}{\mathcal{H}} \right] + \cdots
\end{aligned}
\tag{E-6}
$$

where the terms under the $d\tau'$ integral are evaluated at τ' while the terms in the square braces on the second line are evaluated at τ. The $\cdots$ denotes constant terms evaluated at $\tau = \tau_i$ which arise from the integration by parts. Since our interest is in the preheating phase during which the fluctuations $\delta^{(1)}\sigma$ are amplified exponentially we can safely drop these constant terms.

The second contribution to the last line of (E-4) has the form

$$
-\frac{\phi_k'^{(2)}}{\epsilon \mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2 \phi_k^{(2)}}{3\epsilon \mathcal{H}^2} \ni \frac{1}{6\epsilon} \int_{\tau_i}^{\tau} d\tau' \frac{k^2 J_k^\sigma}{\mathcal{H}(\tau')^3}.
$$

In evaluating this we need only consider terms in $k^2 J_k^\sigma$ which are not suppressed on large scales. Using (E-5) one may verify that

$$
k^2 J_k^\sigma \cong -\gamma_{\sigma,k}'' + \mathcal{H}\gamma_{\sigma,k}'
\tag{E-7}
$$

on large scales (recall that Υ_2^σ is a gradient, see (B-6)) so that we have

$$
\begin{aligned}
- \frac{\phi_k'^{(2)}}{\epsilon \mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2 \phi_k^{(2)}}{3\epsilon \mathcal{H}^2} &\ni \frac{1}{6\epsilon} \int_{\tau_i}^{\tau} d\tau' \frac{k^2 J_k^\sigma}{\mathcal{H}(\tau')^3} \\
&\cong \frac{1}{\epsilon} \int_{\tau_i}^{\tau} d\tau' \left[-\frac{2}{3} \frac{\Upsilon_3^\sigma}{\mathcal{H}} + 6\triangle^{-1}\Upsilon_2^\sigma \right] \\
&+ \left[-\frac{1}{6\epsilon} \frac{\gamma_\sigma'}{\mathcal{H}^3} - \frac{1}{3\epsilon} \frac{\gamma_\sigma}{\mathcal{H}^2} + 2\frac{\triangle^{-1}\Upsilon_2^\sigma}{\mathcal{H}} \right] + \cdots
\end{aligned}
\tag{E-8}
$$

Equation (E-7) shows why it was necessary to include the k^2 terms in the large scale expansion of the Green function (E-4), noting that γ_σ may be written as (B-12) and comparing to (E-5) we see that $k^2 J_k$ contains terms which are of the same size as those in J_k, on large scales. Thus a consistent large scale expansion of $\zeta_\sigma^{(2)}$ requires that we work up to order k^2 in the expansion of the Green function.

The third contribution to the last line of (E-4) is

$$
\begin{aligned}
- \frac{\phi_k'^{(2)}}{\epsilon \mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2 \phi_k^{(2)}}{3\epsilon \mathcal{H}^2} &\ni -\frac{5}{6\epsilon} \int_{\tau_i}^{\tau} d\tau' \frac{k^2 J_k^\sigma}{\mathcal{H}(\tau')\mathcal{H}(\tau)^2} \\
&\cong \frac{5}{6\epsilon} \left[\frac{\gamma_\sigma'}{\mathcal{H}^3} \right] + \cdots
\end{aligned}
\tag{E-9}
$$

using the same procedure as above.

Finally, the fourth contribution to the last line of (E-4) is

$$
\begin{aligned}
- \frac{\phi_k'^{(2)}}{\epsilon \mathcal{H}} - \frac{\phi_k^{(2)}}{\epsilon} - \frac{k^2 \phi_k^{(2)}}{3\epsilon \mathcal{H}^2} &\ni \frac{2}{3\epsilon} \int_{\tau_i}^{\tau} d\tau' \frac{k^2 J_k^\sigma}{\mathcal{H}(\tau)^3} \\
&\cong -\frac{2}{3\epsilon} \frac{1}{\mathcal{H}(\tau)^3} \int_{\tau_i}^{\tau} d\tau' \mathcal{H}(\tau')^2 \Upsilon_3^\sigma \\
&+ \frac{1}{\epsilon} \left[2\frac{\triangle^{-1}\Upsilon_2^\sigma}{\mathcal{H}} + \frac{2}{3} \frac{\gamma_\sigma}{\mathcal{H}^2} - \frac{2}{3} \frac{\gamma_\sigma'}{\mathcal{H}^3} \right] + \cdots
\end{aligned}
\tag{E-10}
$$

Summing up (E-6), (E-8), (E-9) and (E-10) and inserting the result into (E-3) gives

$$
\begin{aligned}
\zeta_\sigma^{(2)} \cong \frac{1}{\epsilon} \int_{\tau_i}^{\tau} d\tau' &\left[-\frac{\Upsilon_1^\sigma}{\mathcal{H}(\tau')} + \frac{1}{3} \frac{\Upsilon_3^\sigma}{\mathcal{H}(\tau')} \right. \\
&\left. - \frac{2}{3} \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3} \Upsilon_3^\sigma \right]
\end{aligned}
$$

Now, using equations (B-3) and (B-9) we can write this in terms of the tachyon fluctuation $\delta^{(1)}\sigma$ as

$$
\begin{aligned}
\zeta_\sigma^{(2)} \cong \frac{\kappa^2}{\epsilon} \int_{-1/a_i H}^{\tau} d\tau' &\left[\frac{\left(\delta^{(1)}\sigma' \right)^2}{\mathcal{H}(\tau')} - \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3} \left(\left(\delta^{(1)}\sigma' \right)^2 \right. \right. \\
&\left. \left. - a^2 m_\sigma^2 \left(\delta^{(1)}\sigma \right)^2 \right) \right]
\end{aligned}
\tag{E-11}
$$

APPENDIX F: The Adiabatic Approximation

Here we consider the construction of curvature perturbation in the case in which the tachyon mass varies adiabatically and demonstrate explicitly that the resulting bispectrum is scale invariant. If the inflaton rolls slowly enough then the tachyon mass will evolve in time extremely slowly and will remain close to zero for a significant number of e-foldings. We define the slow-roll parameter for the σ field as

$$
\eta_\sigma = 4M_p^2 \frac{m_\sigma^2}{\lambda v^4} = -8\eta \left(\frac{M_p}{v} \right)^2 N
\tag{F-1}
$$

For example, if $N_* \sim 30$, then $|\eta_\sigma|$ has the same value at the beginning and end of inflation. We will denote the magnitude of the value of η_σ at the end of inflation (and preheating), $t = t_f$, by

$$
\eta_f = |\eta_\sigma(t_f)|
\tag{F-2}
$$

The condition to have a slowly rolling tachyon during $2N_*$ e-foldings of inflation is therefore

$$
\begin{aligned}
\eta_f &< 1 \\
\implies m_\varphi^2 &< \frac{\lambda v^6}{32 N_* M_p^4}
\end{aligned}
\tag{F-3}
$$

Combining this with the condition (24), it follows that the coupling g must be extremely small if $N_* \gtrsim 1$,

$$
g < \frac{10^{-4}}{N_*} \frac{v}{M_p}
\tag{F-4}
$$

Note that a small coupling g does not require fine tuning in the technical sense, since g^2 is only multiplicatively renormalized: $\beta(g^2) \sim O(g^2\lambda, \ g^4)/(16\pi^2)$. That is, if g is small at tree level, loop corrections will not change its effective value significantly. The bound (F-4) insures that the tachyon will remain light compared to the Hubble scale during N_* e-foldings of inflation.

The combination of (F-4) and (27), eliminating g, gives an upper limit the tachyonic mass scale required for the consistency of our assumptions,

$$\lambda v^2 < \frac{4 \times 10^{-7}}{N_*} M_p^2 \qquad \text{(F-5)}$$

We now turn to the adiabatic approximation for the tachyon mode functions. In this limit the tachyon mass is varying adiabatically, so we can determine the behavior of the fluctuations from the deSitter space solutions by replacing the dependence on a constant tachyon mass with time-varying one. The deSitter solutions for massive fields are given in appendix A. We are interested in the long-wavelength growing modes, which behave like

$$\delta^{(1)}\sigma(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{H}{\sqrt{2k^3}} (-k\tau)^{\eta_\sigma} e^{ikx} a_k + \text{h.c.} \quad \text{(F-6)}$$

in conformal time. Since η_σ is slowly varying, we can replace it with its time-dependent value (F-1), that is

$$\eta_\sigma(\tau) = 8\eta \frac{M_p^2}{v^2} \ln \frac{a_*}{a(\tau)} \qquad \text{(F-7)}$$

Note that $\ln(a_*/a(\tau_f)) = -N_*$ at the end of inflation, so $\eta_f = 8\eta N_* M_p^2/v^2$. We can use (30), which is the criterion for the end of tachyonic preheating and inflation, to determine N_* in terms of the parameters of the hybrid inflation model. Using (F-6), the dispersion can be computed at the end of inflation, $\tau = \tau_f$

$$\begin{aligned}\langle \delta^{(1)}\sigma^2(x) \rangle &= \frac{H^2}{4\pi^2} \int_{k_0}^{k_f} \frac{dk}{k} (-k\tau)^{2\eta_\sigma} \qquad \text{(F-8)} \\ &= \frac{H^2}{8\pi^2\eta_f} \left(|k_0\tau_f|^{-2\eta_f} - |k_f\tau_f|^{-2\eta_f} \right),\end{aligned}$$

where $\eta_f = |\eta_\sigma(\tau_f)| > 0$. The infrared and ultraviolet cutoffs are defined in the same way as is needed to make the dispersion of the scale-invariant inflaton fluctuations finite:[9] $|k_0\tau_i| = 1$ at horizon crossing of the largest scale modes visible today, so that $|k_0\tau_f| = e^{-N_e}$ with $N_e \sim 60$ being the total number of e-foldings of the visible part of inflation. Similarly $|k_f\tau_f| = 1$ at the end of inflation: k_f represents the last mode to cross the horizon before the end of inflation. Even though the integral converges in

---

[9] The first order inflaton contribution to the curvature perturbation's two-point function has the form $\langle(\zeta_\varphi^{(1)}(x))^2\rangle \sim \int d^3k/k^3$, and also requires cutoffs

the absence of the cutoff, later on we will need to know the correct behavior of (F-8) when η_f is small, which is not correctly represented by the limit $k_{max} \to \infty$.

Eq. (F-8) can only describe the growth of the tachyonic fluctuations until eq. (30) is fulfilled, and the perturbative description breaks down. This signals the end of inflation, and allows us to determine the number of e-foldings during preheating, N_*, in terms of the fundamental parameters of the theory. Using (F-8) and (F-1), the implicit relation

$$N_* \cong \frac{\lambda v^6}{64 M_p^4 m_\phi^2} \ln \left[1 + 768\pi^2 N_* \frac{M_p^6 m_\varphi^2}{\lambda^2 v^8} \right] \qquad \text{(F-9)}$$

follows. Using the COBE normalization (24) to eliminate m_φ, this can be rewritten in the form

$$N_* \cong \frac{\sqrt{\lambda}v/M_p}{15000 \, N_e \, g\sqrt{\lambda}} \ln \left[1 + 2 \times 10^6 \, N_* g\sqrt{\lambda} \left(\frac{M_p}{\sqrt{\lambda}v} \right)^3 \right] \qquad \text{(F-10)}$$

This can be solved iteratively on the computer; using an arbitrary guess for N_* and repeatedly evaluating (F-10) gives a rapidly converging answer.

For consistency, we should check whether we have the freedom to make $N_* \gtrsim 1$ given other restrictions on the parameters. From (27), $g\sqrt{\lambda} > 230(\sqrt{\lambda}v/M_p)^3$, and taking $N_e \sim 60$, we get an upper limit on N_*,

$$N_* \lesssim 10^{-7} \frac{M_p^2}{\lambda v^2} (1 + 0.05 \ln N_*) \qquad \text{(F-11)}$$

This would pose a problem for our scenario if it was not possible to take λv^2 to be sufficiently small, say $\lambda v^2/M_p^2 \lesssim 10^{-8}$. However such a value is consistent with the restriction (F-5), so there is no new constraint coming from (F-8). This demonstrates that it is consistent to assume that the tachyon remains light until the end of inflation, provided that g is sufficiently small, eq. (F-4).

We now perform the time integral in the result (57) using the knowledge of how $\delta^{(1)}\sigma$ behaves in the long-wavelength limit which will be of interest for cosmological observables, (F-6). According to this result, $\delta^{(1)}\sigma' \sim (\eta_\sigma/\tau)\delta^{(1)}\sigma$, whereas $m_\sigma^2 a^2 \cong m_\sigma^2/(H^2\tau^2) = 3\eta_\sigma$. Therefore, since we are working in the regime where the tachyon is light, $\eta_\sigma < 1$, the term with m_σ^2 in (57) dominates over the terms with time derivatives. Let τ_f be the final time in the upper limit of integration in (57), and τ_* the time at which the tachyonic instability begins. For concreteness, we will at first assume that there are equal numbers of e-foldings ($N_* \sim 30$) before and after this time, so that the slow-roll parameter for the tachyon is time-antisymmetric, $\eta_\sigma(\tau_i) = -\eta_\sigma(\tau_f)$, since this is the way of having a light tachyon during inflation which minimizes the tuning of the tachyon mass. Changing variables to $x = -\ln(\tau'/\tau_*)$, and writing $\eta_\sigma = -dx$ with $d = \eta M_p^2/v^2$, eq. (57) in Fourier space takes the

form

$$\zeta_\sigma^{(2)}(k) \cong 3d\frac{\kappa^2}{\epsilon} \int \frac{d^3p}{(2\pi)^{3/2}} (\delta^{(1)}\hat{\sigma})_p (\delta^{(1)}\hat{\sigma})_{k-p} \quad \text{(F-12)}$$

$$\int_{-N_*}^{N_*} dx\, |p\tau_*|^{-dx}|(k-p)\tau_*|^{-dx} e^{2dx^2+3(x-N_*)} + O(d^2)$$

where $\delta^{(1)}\hat{\sigma}_p$ is the part of $\delta^{(1)}\tilde{\sigma}_p$ left after dividing by the only time-dependent factor, $(-p\tau)^{\eta_\sigma}$. In fact $\delta^{(1)}\hat{\sigma} \sim H/(2p^3)^{1/2}$ is the perfectly scale-invariant part of the perturbation, as far as its contribution to the power spectra of fluctuations, while the $(-p\tau)^{\eta_\sigma}$ gives rise to the small departures from scale invariance.

The integral (F-12) is dominated by contributions near the final time, so one expects it to behave roughly like the value of the integrand at $x = N_*$. By numerically evaluating (F-12) and studying its dependences on N_*, d and $k\tau_*$, we are able to deduce the following semi-analytic fit:

$$\zeta_\sigma^{(2)} \cong \frac{\kappa^2}{\epsilon} \frac{\eta_f}{1+\eta_f} e^{2\eta_f N_*} \int \frac{d^3p}{(2\pi)^{3/2}} (\delta^{(1)}\hat{\sigma})_p (\delta^{(1)}\hat{\sigma})_{k-p}$$
$$|p\tau_*|^{-\eta_f} |(k-p)\tau_*|^{-\eta_f} \quad \text{(F-13)}$$

where $\eta_f = |\eta_\sigma(\tau_f)|$ is the magnitude of the final value of η_σ. Notice that $|p\tau_*| = p/(a_*H)$, so that a mode which crossed the horizon at the beginning of inflation (satisfying $|p\tau_i| = 1$) would have $|p\tau_*| = e^{-N_*}$. The second order perturbation at these scales therefore grows by a factor of $e^{4\eta_f N_*}$ relative to scale-invariant perturbations, assuming $|k-p| \sim p$. This is the growth which is due to the tachyonic instability. We remind the reader that this growth saturates and our perturbative treatment breaks down when $\delta^{(1)}\sigma \sim v/2$, whereas initially $\delta^{(1)}\sigma \sim H$, so the result (F-13) is only valid when $e^{4\eta_f N_*} \lesssim v/H$.

We can also consider a different case, when horizon crossing occurs at the time τ_*, and then $N_* \sim 60$ instead of 30. We find that the formula (F-13) is essentially unmodified, because of the fact that the integral is dominated by its late time behavior, so that the contributions from the period $\tau < \tau_*$ are negligible. In this case, the total growth in $\zeta_\sigma^{(2)}$ comes from the factor $e^{2\eta_f N_*}$ and not from $|k\tau_*|$, since the latter is of order unity. Nevertheless the total amount of growth in the two cases is the same, $\sim (e^{60\eta_f})^2$, and its range of applicability is limited by the same factor $(v/H)^2$.

We now calculate the leading contribution to the bispectrum of the second-order curvature perturbation (61). Since there is no momentum dependence in the result (F-13) for $\zeta^{(2)}$, this three-point function is straightforward to compute, using the massless free-field two-point functions

$$\left\langle \delta^{(1)}\hat{\sigma}_{p_i} \delta^{(1)}\hat{\sigma}_{q_i} \right\rangle = \frac{H^2}{\sqrt{2|p_i|^3\, 2|q_i|^3}} \delta^{(3)}(p_i + q_i) \quad \text{(F-14)}$$

Carrying out the contractions of pairs of fields which contribute to the connected part of the bispectrum, one

finds eight terms, which are compactly expressed in terms of the wave numbers p_i and $q_i = k_i - p_i$:

$$\left\langle \zeta_{k_1}^{(2)} \zeta_{k_2}^{(2)} \zeta_{k_3}^{(2)} \right\rangle_{\text{con}} = (2\pi)^{-9/2} \int \prod_i d^3p_i\, f(p_i,k_i)$$
$$\times \Big[\delta_{p_1+p_2}(\delta_{q_2+p_3}\delta_{q_3+q_1} + \delta_{q_2+q_3}\delta_{p_3+q_1}) \quad \text{(F-15)}$$
$$+ \delta_{p_1+q_2}(\delta_{p_2+q_3}\delta_{p_3+q_1} + \delta_{p_2+p_3}\delta_{q_3+q_1}) + (p_i \leftrightarrow q_i) \Big]$$

where

$$f(p_i,k_i) = d_*|\tau_*|^{-2\eta_f} |p_i|^{-3/2-\eta_f} |k_i - p_i|^{-3/2-\eta_f} \quad \text{(F-16)}$$

and $d_* = H^2\kappa^2\eta_f e^{2\eta_f N_*}/(2\epsilon)$. Each triplet of delta functions contains the factor $\delta_{k_1+k_2+k_3}$ expressing the translational invariance of the bispectrum. The remaining two delta functions can be used to perform the integrals over p_1 and p_2, leaving the $p_3 \equiv p$ integral.

The resulting bispectrum can be shown (with help from Maple) to reduce to the sum of two terms,

$$B(\vec{k}_i) = 4|\tau_*|^{-6\eta_f} d_*^3 \int \frac{d^3p}{(2\pi)^3} |p|^{-3-2\eta_f} |p-k_3|^{-3-2\eta_f}$$
$$\left(|p+k_2|^{-3-2\eta_f} + |p+k_1|^{-3-2\eta_f} \right) \quad \text{(F-17)}$$

This expression is manifestly symmetric under $k_1 \leftrightarrow k_2$, but it can also be shown to be symmetric under interchange of any two k_i's by an appropriate shift of the integration variable. The integral converges as $p \to \infty$, but since $\eta_f > 0$ by definition, it diverges with the small power $1/p^{\eta_f}$ near $p = 0$, and we must introduce an infrared cutoff, k_0, representing a scale a few e-foldings beyond our present horizon. This procedure was also necessary for the dispersion of the fluctuations $\langle (\delta^{(1)}\sigma)^2 \rangle$ in eq. (F-8). But unlike in (F-8), we must cut off the integral not just for $|p| < k_0$, but also for $|p - k_3| < k_0$, $|p+k_1| < k_0$, and $|p+k_2| < k_0$. The physical reasoning is however the same: there should be no observable effects coming from fluctuations whose wavelength is far beyond our present horizon.

By dimensional analysis, one can see that if all wavenumbers have the same magnitude, $k_i = k\hat{k}_i$, then the bispectrum goes like $k^{-6(1+\eta_f)}|\tau_*|^{-6\eta_f}$ times a dimensionless function of the directions, $\hat{k}_i \cdot \hat{k}_j$, of k/k_0 and of η_f. For simplicity, we evaluate it at the symmetric point $\hat{k}_1 = (-1/2, \sqrt{3}/2, 0)$, $\hat{k}_2 = (-1/2, -\sqrt{3}/2, 0)$, $\hat{k}_3 = (0, 1, 0)$, where the three wave vectors form an equilateral triangle. The integral is evaluated numerically to obtain the result

$$B(\vec{k}_i) \cong \frac{1}{k^6} f_3(k/k_0, \eta_f) \left(\frac{H^2\kappa^2\eta_f e^{2\eta_f N_*}}{2\pi\, \epsilon |k\tau_*|^{2\eta_f}} \right)^3 \text{(F-18)}$$

We plot the function $f_3(k/k_0, \eta_f)$ in figure 15, which shows strong sensitivity to the presence of the cutoff for modes with $k \lesssim k_0$, but weak scale-dependence for modes just a few e-foldings shorter in wavelength. Since observable quantities should not be sensitive to the exact value

of the cutoff, the region with $\ln(k/k_0) > 1$ is the physically relevant one. In this region, a reasonable fit to the result for $\eta_f \lesssim 0.1$ is given by

$$f_3 \sim 45 \left(\frac{k}{k_0}\right)^{0.35} \qquad \text{(F-19)}$$

which is also plotted in figure 15. It is clear, then, that in the case of an adiabatically varying tachyon mass there is only a small breaking of scale invariance. This breaking of scale invariance can be neglected when evaluating the bispectrum near $k = k_0$, corresponding to the largest presently observable scales. Our infrared cutoff is effectively at wavelengths a few e-foldings larger than this scale.
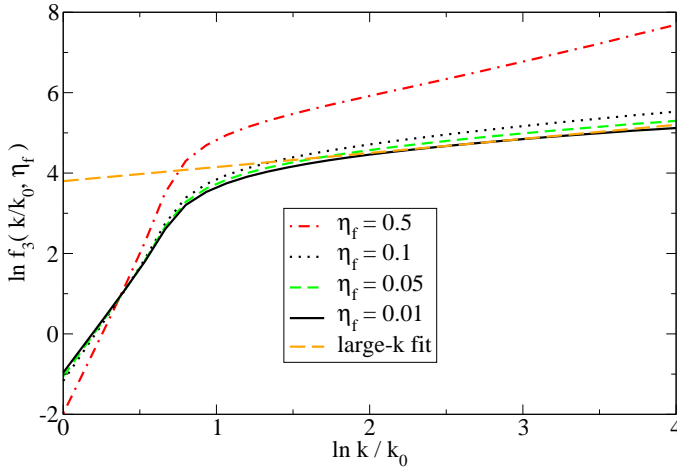

FIG. 15: Log of $f_3(k/k_0, \eta_f)$ versus $\ln(k/k_0)$ for several values of η_f, and the large-k fit (F-19).

We have computed the nonlinearity parameter f_{NL} using the approach detailed in this appendix and have found that large nongaussianity is produced for all values of the parameters which are consistent with the assumptions we have made.

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